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COLLEGE OF ENGINEERING AND PHYSICAL SCIENCES

The Real Story of the Comet Disaster

De Havilland Comet – Structural Fatigue

Paul Withey

Lecture organised by RAeS Hamburg Branch
Hamburg Aerospace Lecture Series
(DGLR, RAeS, VDI, ZAL, HAW Hamburg)
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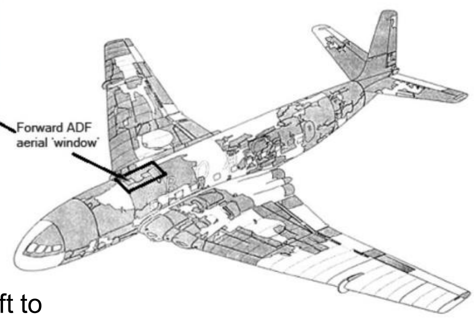
The Real Story of the Comet Disasters

Prof. **Paul Withey,** School of Metallurgy and Materials, University of Birmingham

Date: Thursday 24 January 2019, 18:00
Location: HAW Hamburg Berliner Tor 5, (Neubau), Hörsaal 01.11

Lecture followed by discussion
No registration required !
Entry free !



The de Havilland Comet was the first commercial jet aircraft, and ushered in the 'Jet Age' on 2nd May 1952 by taking fare paying passengers from London to Johannesburg.
This aircraft contained a number of new technologies to allow the aircraft to operate economically and to enhance the flying experience for the passengers. For a number of months the aircraft led the world by halving journey times and offering comfort levels which could not be matched on other, piston engine aircraft. However, two accidents in 1954 grounded the Comet fleet and the subsequent investigation has ensured the Comet has notoriety as an example of fatigue failure. This high profile incident encouraged much work in the field of fatigue and this has led to a much better understanding of the science of fatigue and the use of fracture mechanics to evaluate the life of components and structures.
This talk will look at the history of the Comet aircraft, from concept to entry into service, review the accident investigation and use modern analysis to review the fatigue failure which sparked the research. Using this analysis the general perceptions of the causes can be examined and a likely chain of events which led to the failure is proposed.

Paul Withey joined the University of Birmingham School of Metallurgy and Materials in 2018 after a career in Rolls-Royce, culminating as the Engineering Associate Fellow in Casting Technology. This has limited the number of refereed papers to just over thirty, but this has been balanced by fourteen published patents. Paul's interests involve investment casting with a focus on the casting of aerospace components. Much of his industrial career has been spent developing the processes, and understanding of the single crystal casting of turbine components. The fundamental link between the materials chosen for casting, processing to form the desired shape, and the properties achieved through processing will form the basis of his ongoing research.

DGLR / HAW	Prof. Dr.-Ing. Dieter Scholz	Tel.: (040) 42875 8825	info@ProfScholz.de
DGLR	Dr.-Ing. Martin Spiek	Tel.: (040) 9479 2855	martin.spiek@thelsys.de
RAeS	Richard Sanderson	Tel.: (04167) 92012	events@raes-hamburg.de

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Abstract

Design work began on the De Havilland Comet in September 1946. Key features: 36 passengers, range 2800 km, cruising speed 780 km/h, cruising altitude 35000 ft, aluminium construction, four de Havilland Ghost Jet engines. The pressurised cabin was designed for a cabin pressure equivalent to an altitude of 8000 ft. De Havilland were aware of the likelihood of fatigue and had installed several safety measures and tests in line with certification requirements. First flight was on 27th July 1949. The aircraft entered service on 2nd May 1952 (G-ALYP) and put the UK aircraft industry at the forefront of technology. First accident of a De Havilland Comet was on 10th January 1954. G-ALYP crashed into the sea leaving Rome. Flight services was resumed on 23rd March 1954. The second accident was on 8th April 1954. G-ALYY crashed also into the sea leaving Rome. The Certificate of Airworthiness was withdrawn 12th April 1954. Intensive research followed, concentrating on the understanding of structural fatigue. A Comet fuselage was pressurized in a water tank. The recovered wreckage of the "YP" was assembled on frames by the RAE. It was found that the aircraft disassembled in the air. The accident was caused by structural failure of the pressure cabin, brought about by fatigue. The square windows were the cause of high stresses. The bolt hole which failed on "YP" had a defect in the chamfer which indicated the potential for manufacturing defects on all skin holes. The interaction of the skin stresses and the manufacturing defects was beyond the scientific knowledge base of the early 1950s. The Comet flew again as the Comet IV with different window design. The Comet was the first airliner to fly a scheduled service across the Atlantic on 4th October 1958. It remained in service as the Nimrod until 60 years after first Comet flight. The presentation revisits the Comet case and shows a modern investigation based on the research done in the 1950th and the "Paris Law" from 1963 which allows the calculation of crack growth. Using the data from the 1950's, it was calculated: parameter A = 49.5 MPa m^(1/2) and exponent m = 5. As such, the material behaved slightly worse than current similar alloys. Crack growth analysis calculated the life of "YP" as 1272 cycles. The actual number of pressurised flights was 1290.

(*Abstract written by Dieter Scholz based on the text of the presentation.*)

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AIR FRANCE
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Background - General

- Post war air travel dominated by large propeller driven aircraft
- US Industry developed bespoke airframes
  - E.g. Lockheed Constellation
- UK Industry offered modified wartime transports
  - E.g. Avro York
- Air travel was becoming popular and comfort and speed was being expected

- Military aircraft had been using gas turbine power plants since 1942 but they were the preserve of only a few air forces

Background – de Havilland

- De Havilland had a good pedigree in military jet aircraft
- The Vampire was the second jet fighter to enter service with the RAF in 1945
- Powered by one de Havilland Goblin engine
  - 3500lbs thrust

- Passenger aircraft were also a key part of the de Havilland history
  - DH4A completed the world's first scheduled passenger flight in 1919

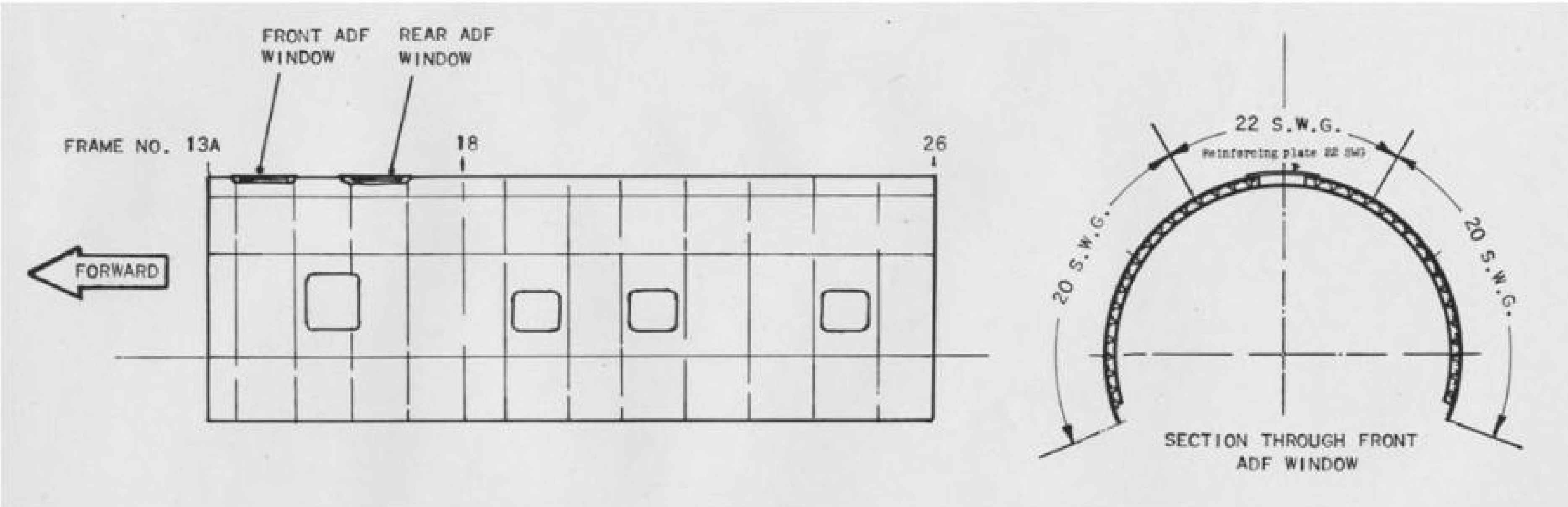
Background – de Havilland Comet

- Based on the requirements of the Brabazon Committee
  - High speed mail plane
- Design work began in September 1946
- Key features
  - Aluminium construction
  - Hydraulic actuation of the control surfaces
  - Four de Havilland Ghost Jet engines (4450lbs thrust each)
  - 36 passengers
  - Range 1750 miles (2800 km)
  - Cruising speed 490 mph (780 kmh^{-1})
  - Cruising altitude 35000 ft (10.6 km)
  - Weight 107,000 lbs (49,000 kg)

Background – Technical Challenges

- Gas Turbine Engines are more efficient at high altitude
  - Passengers would need oxygen
  - Pressurised cabin was designed for passenger comfort
  - Passenger cabin pressurised to 8.25psi (56kPa) – equivalent to an altitude of 8,000’ (2.4km)
- The Ghost engines were not powerful by today’s standard
  - Thrust to weight ratio of around 2
  - Today a comparable engine would be better than 6
  - Airframe was made as light as possible to enable the maximum payload
- Skin panel joining was done by an aluminium ‘glue’ called ‘Redux’
  - Aluminium alloy DTD546 (similar to a 2XXX series alloy)

Background – Technical Detail



Background – Fatigue Crack Failure

- Definition of fatigue
  - "Metals will break under a load which is repeatedly applied and then removed, though they can support a much larger steady load without distress."
- Known about for over 100 years by 1950
  - Railway truck axle failures in the 19th century
  - Military aircraft suffered fatigue during WW2
    - Wing structures were being examined
- De Havilland were aware of the likelihood of fatigue

Background – Legal Testing Requirement

- British Civil Airworthiness Requirements in 1949
  - Proof pressure of $1^1/_3$P (no permanent deformation)
  - Design pressure of 2P
- De Havilland used a higher rating in designing the Comet
  - 2P proof pressure
  - $2^1/_2$P design pressure
- Belief shared by de Havilland and the Air Registration Board that this higher proof stress would protect against fatigue
- Fatigue thresholds in steels were known but not thought to be present in aluminium alloys
- Over designing the structure was an acceptable methodology

Background – Legal Testing Requirement

- Draft Requirements from mid 1952 (Issued June 1953)
  - Static test to 2P
  - Proof test to $1^1/_3$P
  - 15,000 applications of $1^1/_4$P
  - Structural parts need to with stand 3P
    - Doors, riveted joints, window frames etc.
- 15,000 cycles was seen as the life of the airframe
- Application of $1^1/_4$P was to cover the scatter in fatigue results
- Fatigue was still not understood well enough to enable an accurate lifing methodology to be used

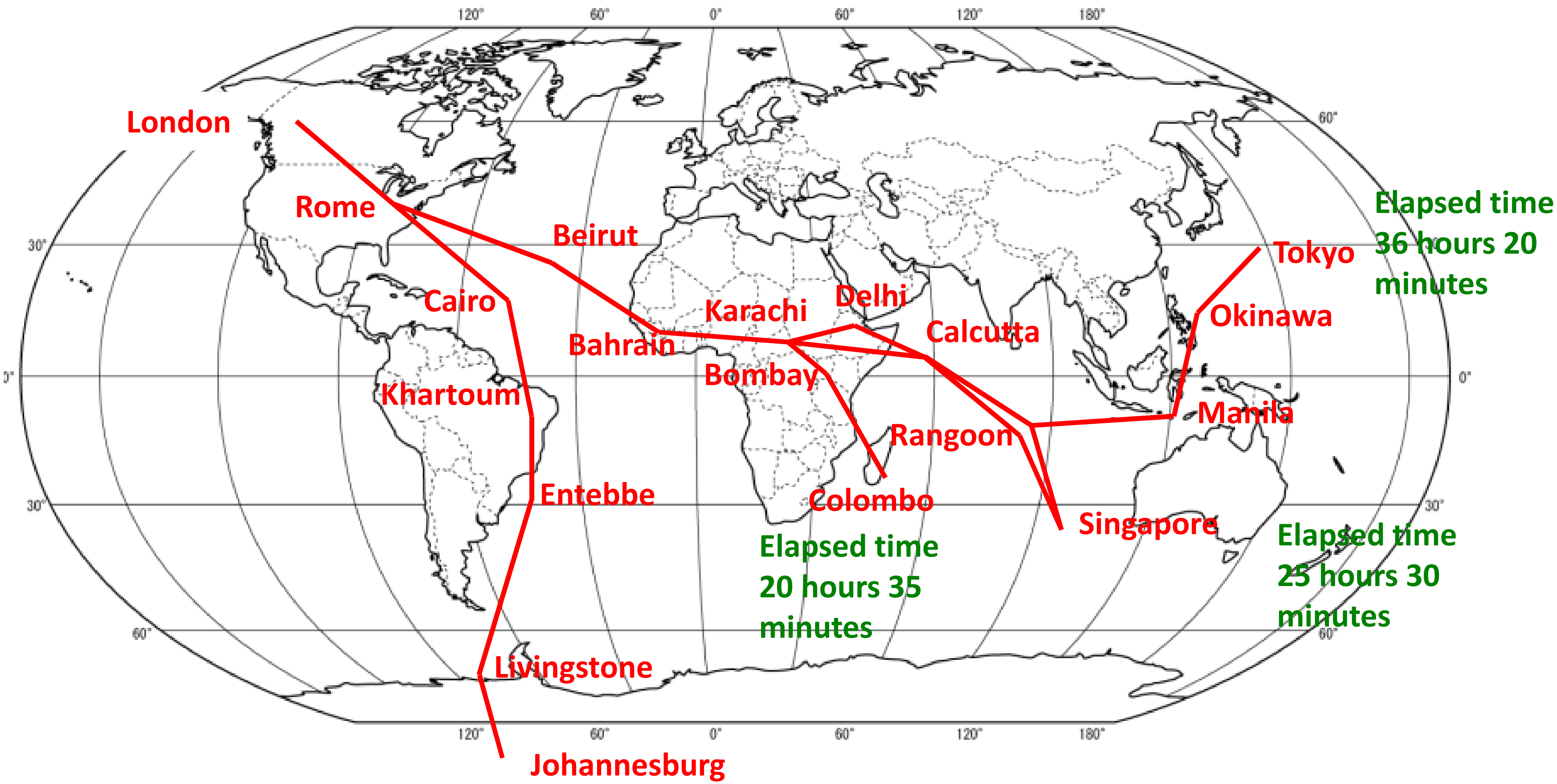
Background – de Havilland Testing

- Two cabin sections were evaluated for fatigue
- Forward section 26’ (7.92m) long
  - Proof tested to twice the operating pressure (2P)
  - Cycled up to the operating pressure (P) 18,000 times
- Mid section 24’ (7.32m) long
  - Proof tested to twice operating pressure (2P)
  - Cycles up to operating pressure (P) 16,000 times
- Exceeded the legal requirements
  - By July 1953 the forward section had seen 16,000 cycles and the flying Comets had not exceeded 2,500 hours service (about 800 cycles)
- Sections including cut outs were proof tested to stresses higher than 2P
  - Preferred real tests to stress calculations

De Havilland Comet

- First flight 27th July 1949
- First public display September 1949 at Farnborough
- Broke many passenger aircraft records
  - London to Rome, Cairo and Copenhagen
- Entered service on 2nd May 1952 (G-ALYP)
  - London to Johannesburg
- Inaugurated a London to Tokyo service in April 1953
  - 1.5 days (half the previous journey time)
  - 89% load factors
- Put the UK aircraft industry at the forefront of technology

De Havilland Comet



Elapsed time 21 hours 20 minutes

One year in:
370 hours per week
122,000 miles
20,780 unduplicated miles

De Havilland Comet - Accidents

- Near Elba (leaving Rome) 10th January 1954
  - G-ALYP
  - Crashed into the sea (1290 pressurised flights)
- All Comets grounded
- Investigation inconclusive due to lack of wreckage
  - Many changes made to the engines and airframe

- "Although no definite reason for the accident has been established, modifications are being embodied to cover every possibility that imagination has suggested as a likely cause of the disaster. When these modifications are completed and have been satisfactorily flight tested, the Board sees no reason why passenger services should not be resumed."
  - Lord Brabazon, Chair of the ARB

- Flight services resumed 23rd March 1954


B O A C

De Havilland Comet - Accidents

- Near Naples (leaving Rome) 8th April 1954
  - G-ALYY
  - Crashed into the sea
- All Comet services suspended by BOAC

- Certificate of Airworthiness withdrawn 12th April 1954

- Minister of Supply instructed Sir Arnold Hall, the Director of the Royal Aircraft Establishment, to "undertake a complete investigation of the whole problem presented by the accidents and to use all the resources at the disposal of the Establishment."

RAE Investigation

- Covered a number of different avenues
  - Rebuild of the wreckage of Comet ‘YP’
  - Fatigue tests on the pressure cabin
  - Fatigue tests on the wings
  - Fatigue tests on the tail plane
  - Static strength of the tail plane
  - Damage to the outer wing tanks during refuelling
  - Possibility of excessive pressures in the fuel tanks and cabin
  - Possibility of loss of control
  - Free flight tests of dynamic models
  - Flight investigation of Comet G-ANAV
  - Miscellaneous investigations
  - Medical aspects of the accident

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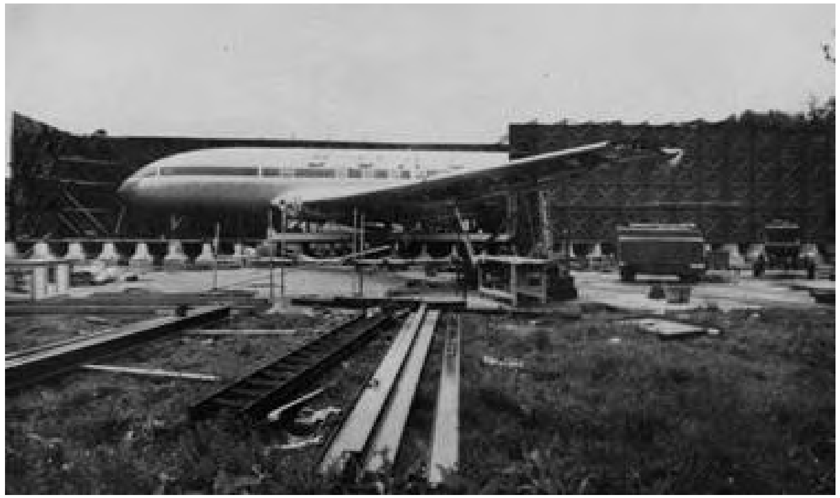
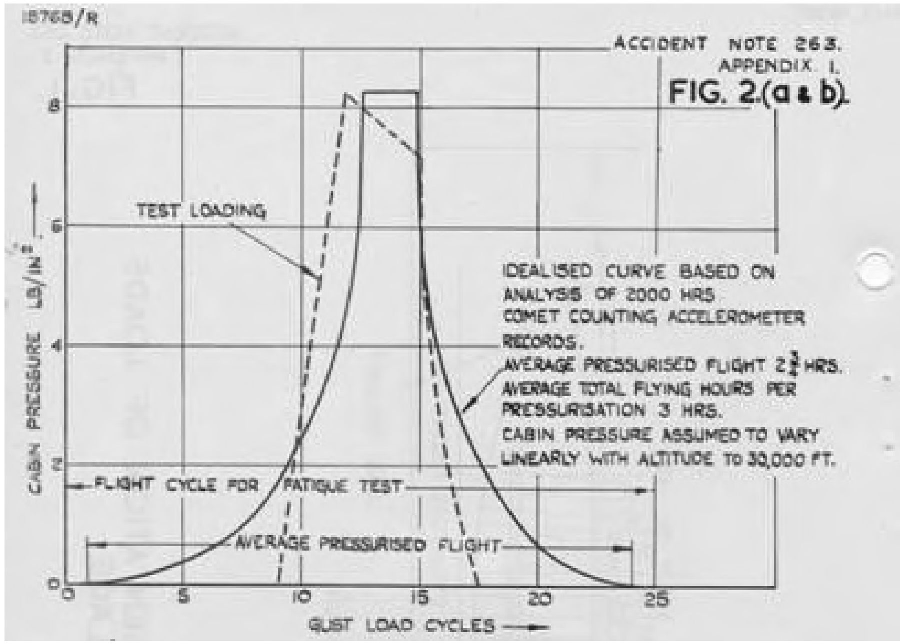
RAE Investigation – Fatigue Tests

- Fatigue tests on the pressure cabin and wings
- Full cabin fatigue test
- Water used to prevent explosive decompression
- Two water tanks built, 218,000 gallon (991,000 litre) each
- Simulated flight cycle loads applied using hydraulic rams
- Aerodynamic loads and gust loads (data from BOAC flights)
- Two hour flight simulated in five minutes

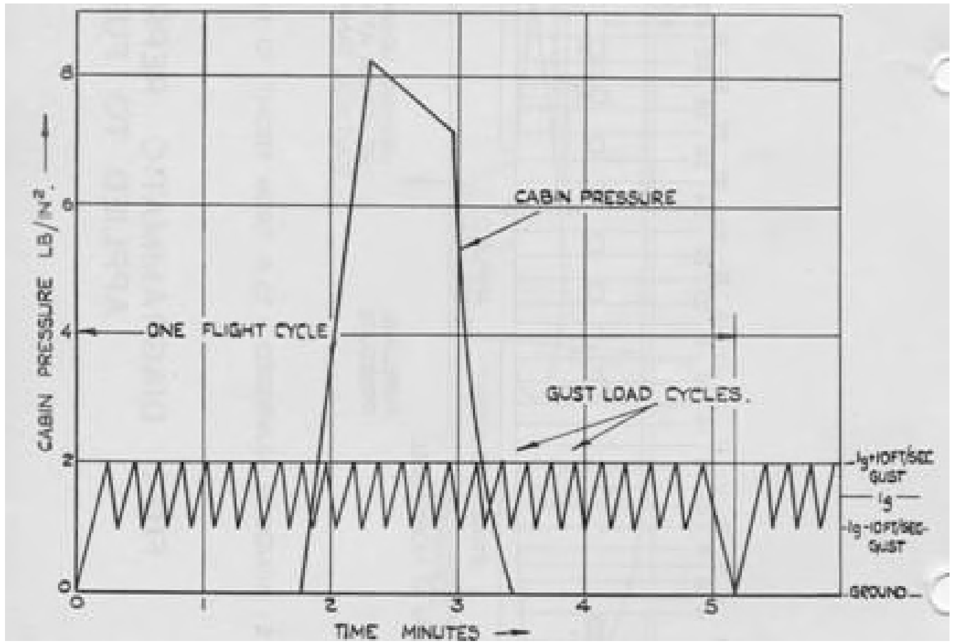


RAE Investigation – Fatigue Tests

Gust and flight loads from BOAC flights

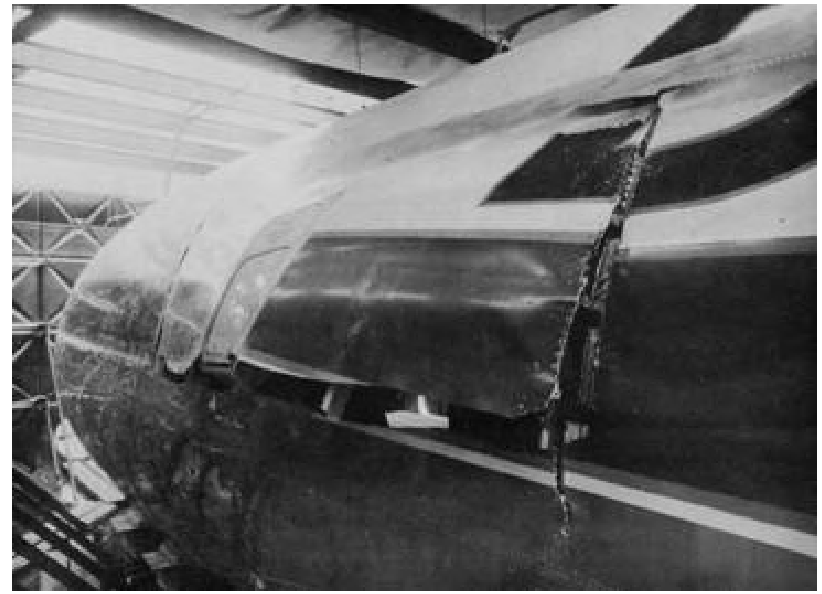


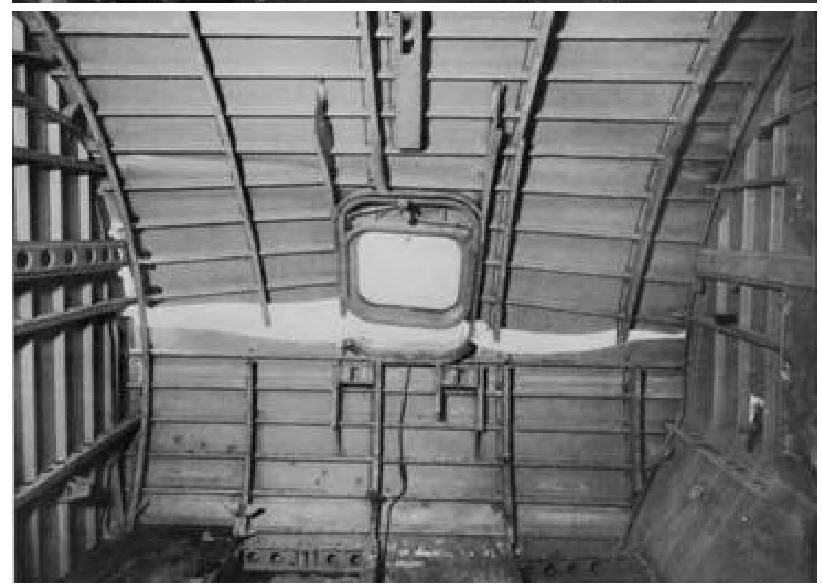
Simulated Cycle



RAE Investigation – Fatigue Tests

- G-ALYU was placed inside the tank
- All internal cabin fixtures were removed and replaced with weights
- ‘YU’ had already completed 1121 Passenger flights
- ‘YU’ had completed 10 pressurised flights with de Havilland plus other tests
- A further 1826 simulated cycles were completed before fatigue failure of the pressure cabin from a crack growing from a rivet hole at the forward port escape hatch
- Total number of cycles completed was 3057





RAE Investigation – Stress Measurements

- Strain gauges used to establish the stresses around cut outs
- Peak stress at the edge of the window was estimated as:
  - 43,100 psi (297MPa) due to cabin pressure
  - 650 psi (4.5 MPa) due to flight loads
  - 1,950 psi (13.5 MPa) due to gusts
  - 45,700 psi (315 MPa) in total
- Peak stress was found to be 70% of the UTS (450 MPa)

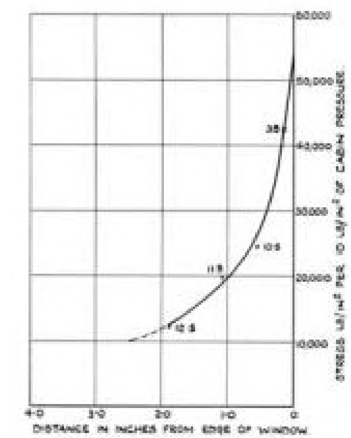
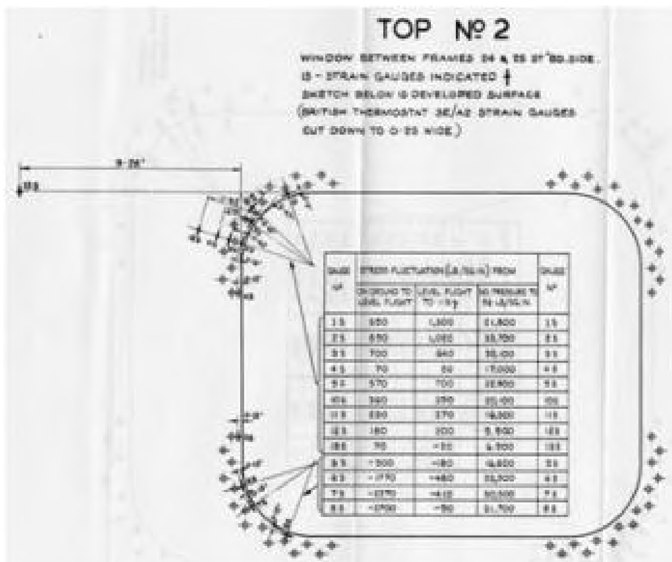


FIG 8 SHOWING BUILD UP OF STRESS TOWARDS THE TOP AFT CORNER OF Nº 2 WINDOW AT 10 LB/IN² CABIN PRESSURE

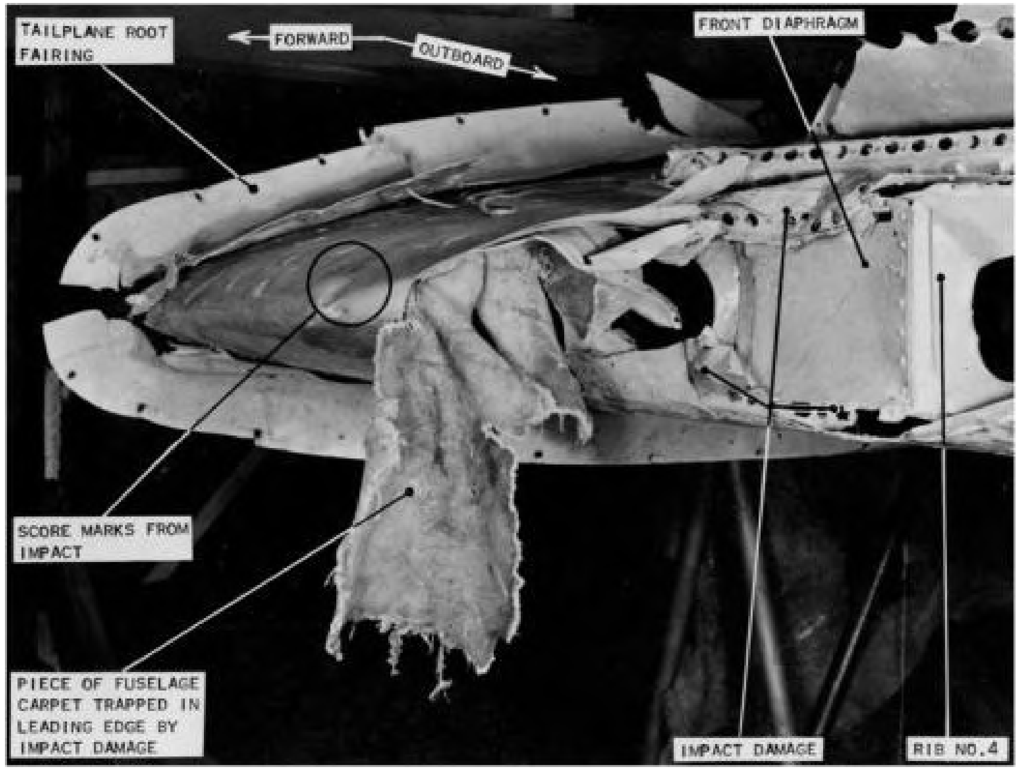


RAE Investigation – Re Build of ‘YP’

- ‘YP’ crashed into the sea in 600’ of water (183m)
  - The Royal Navy gathered as much of the wreckage as they could find
  - 70% of the aircraft was located by September 1954
- ‘YY’ was thought to rest at depths up to 3500’ (1060m)
  - Recovery was not possible at these depths
  - Only floating wreckage was recovered
- ‘YP’ was gradually rebuilt
- Similarities between ‘YP’ and ‘YY’ were noted
  - Flight profile was similar
  - Forensic evidence was similar
- Evidence of the breakup sequence of ‘YP’ was seen

RAE Investigation – Re Build of ‘YP’

- The tail plane separated early in the break up sequence
- Damage to the tail plane by pieces from the cabin interior
  - Coin impact
  - Carpet
- Pressure cabin must have failed first

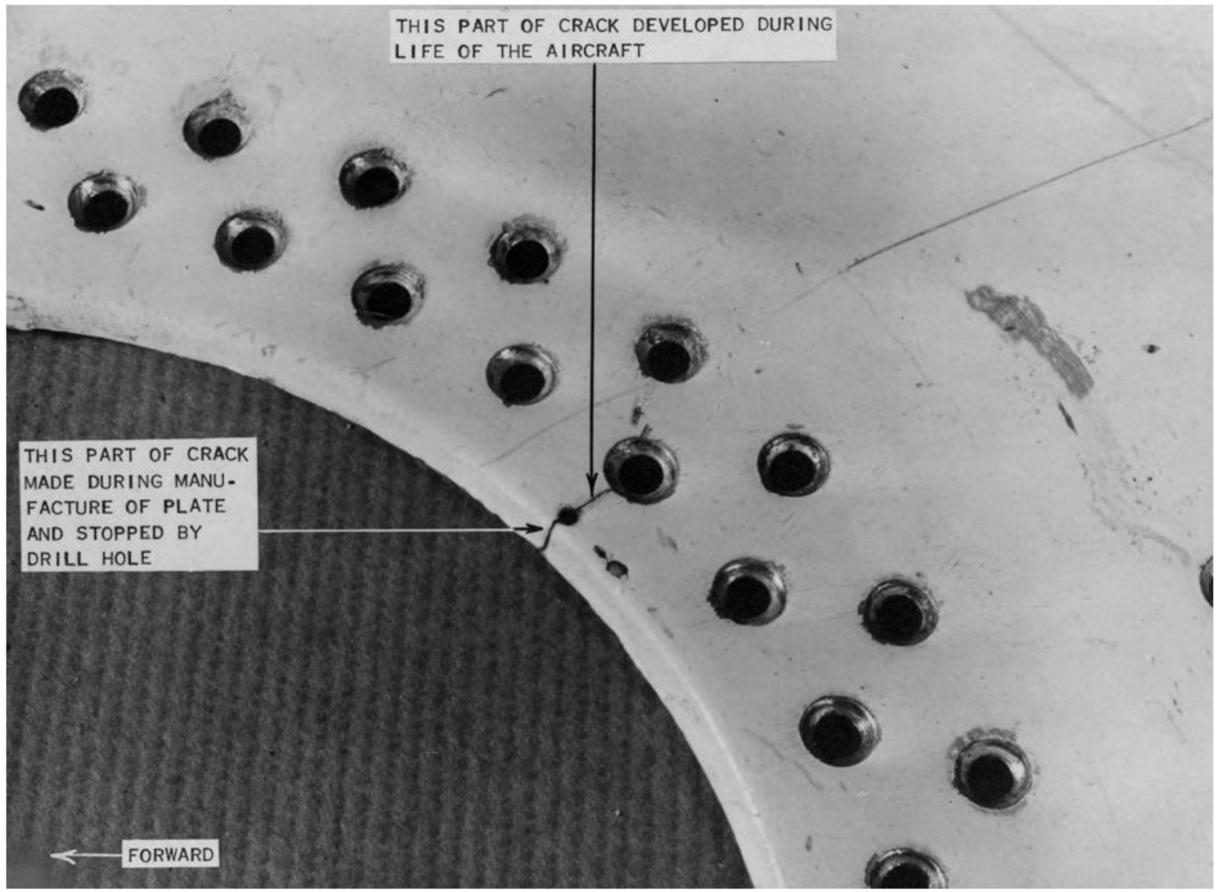


RAE Investigation – Re Build of 'YP'

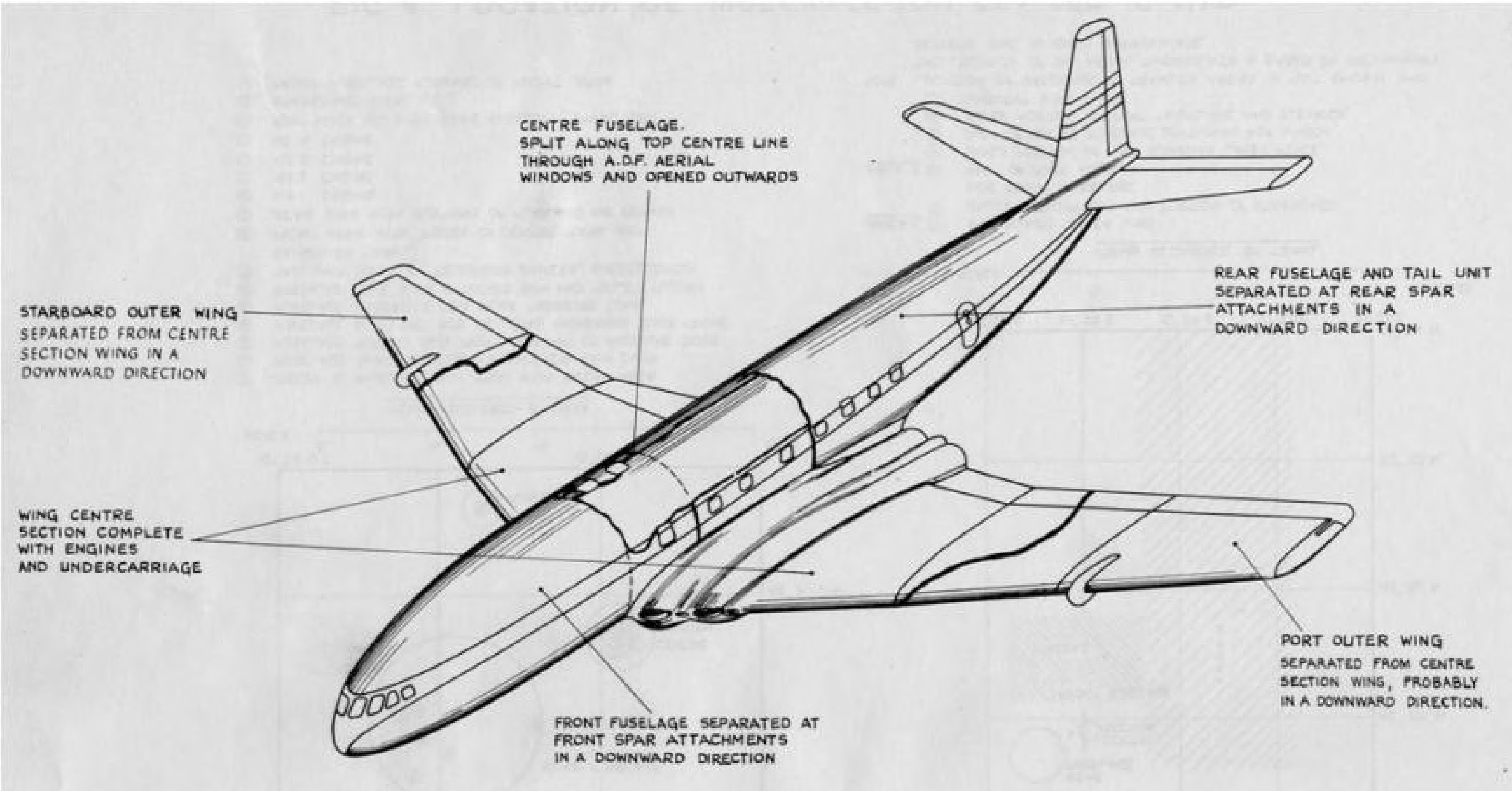
- The recovered wreckage was assembled on frames
- Allowed parts to be inspected in relation to adjacent parts
- Failure traced back to the upper forward portion of the cabin
  - Actual part recovered in August 1954

RAE Investigation – Manufacturing Cracks

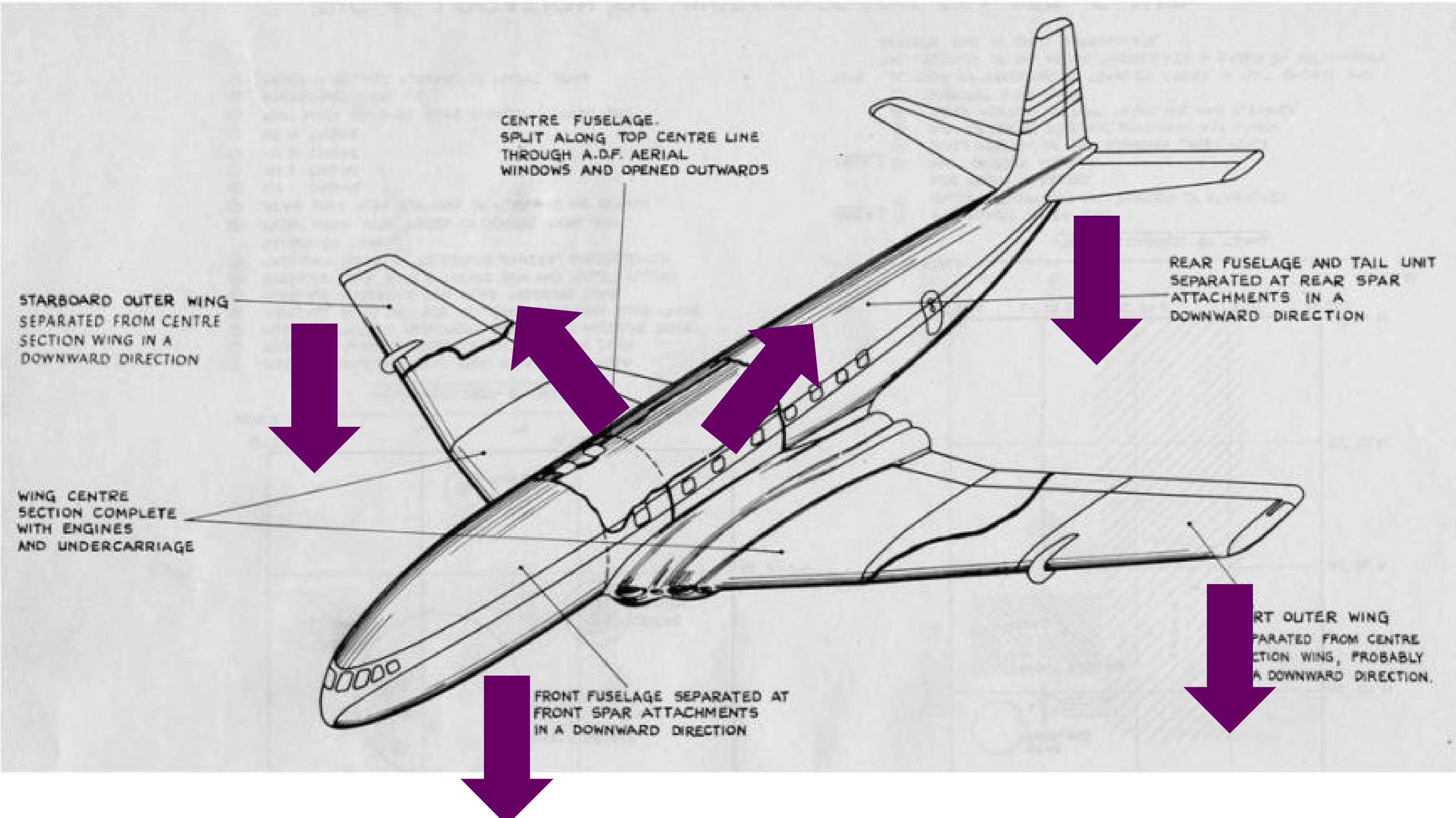
- Manufacturing cracks were found
- Riveting was likely to cause cracks in the skin around the rivet holes
- On inspection these were ‘stopped’ with a 1/16’’ (1.6 mm) drill
- During the investigation some were seen to continue beyond hole others did not



RAE Investigation - Break Up Sequence



RAE Investigation - Break Up Sequence



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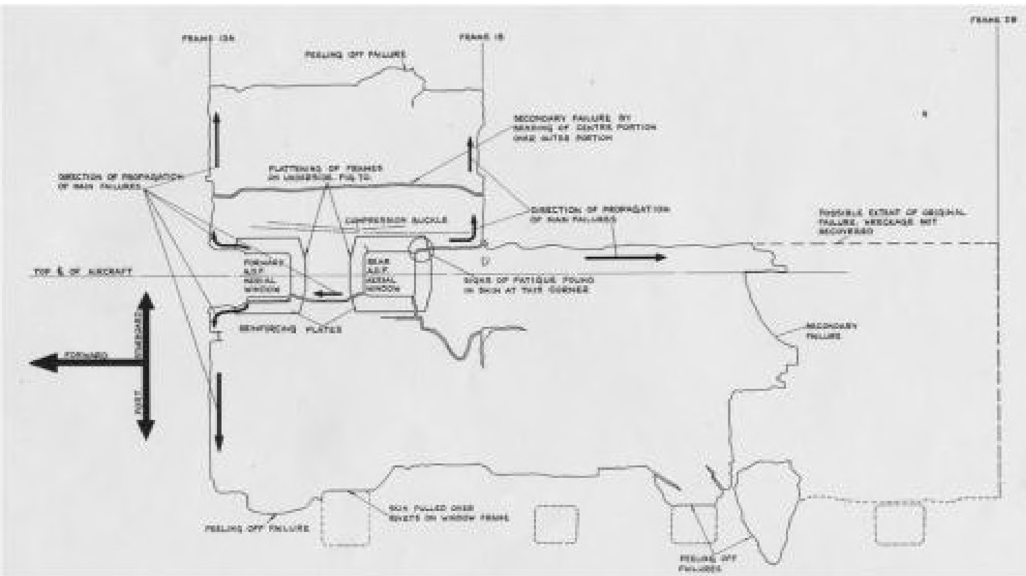
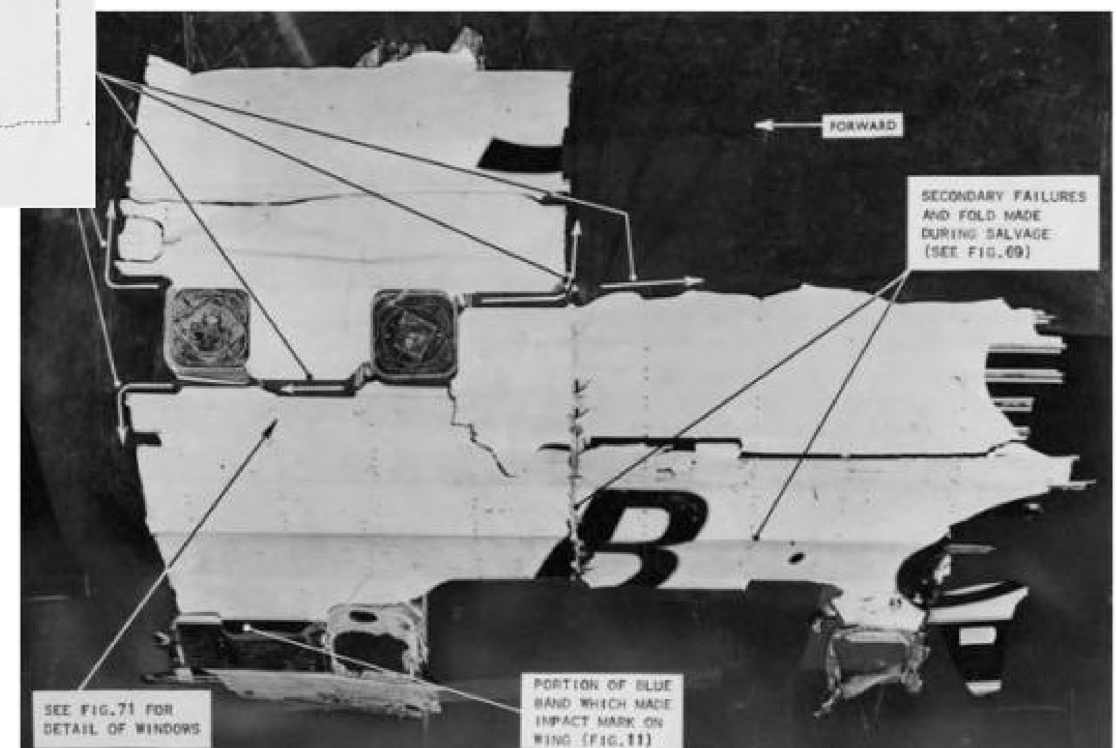
RAE Investigation - Break Up Sequence





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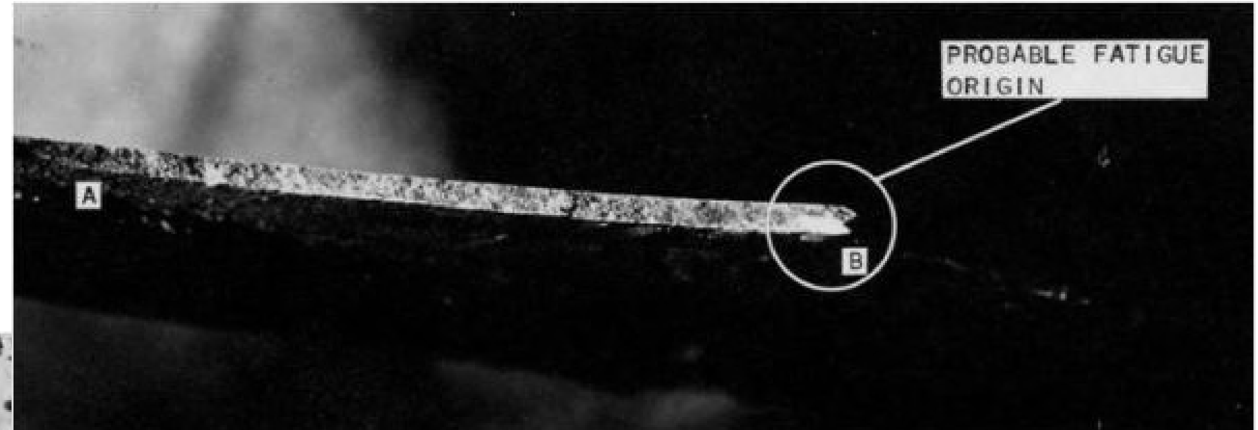

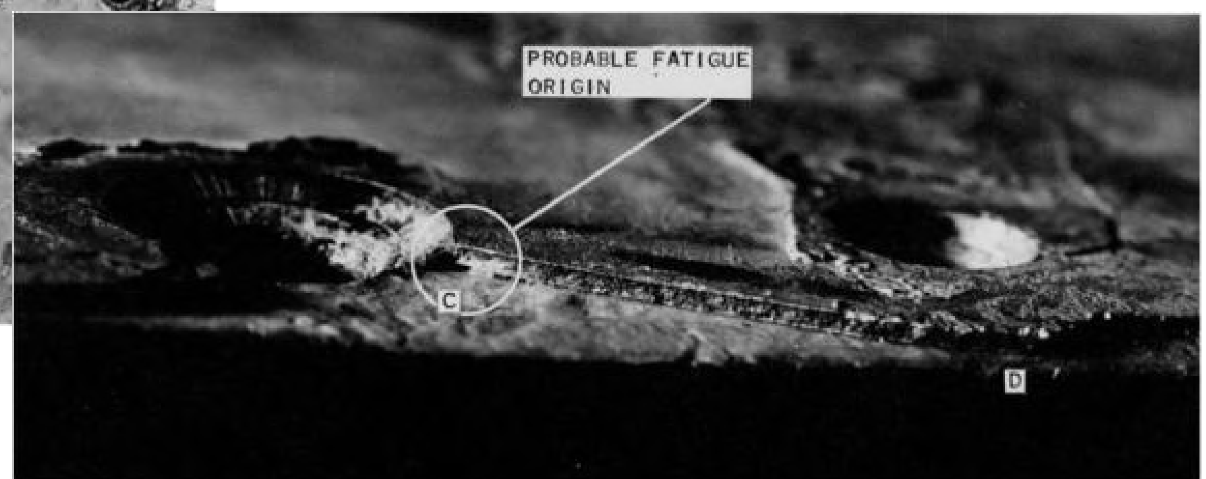

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RAE Opinion

- We have formed the opinion that the accident at Elba was caused by structural failure of the pressure cabin, brought about by fatigue. We reached this opinion for the following reasons:-
  - The low fatigue resistance of the cabin has been demonstrated by the test described in Part 3, and the test result is interpretable as meaning that there was, at the age of the Elba aeroplane, a definite risk of fatigue failure occurring (Part 3).
  - The cabin was the first part of the aeroplane to fail in the Elba accident (Part 2).
  - The wreckage indicates that the failure in the cabin was the of same basic type as that produced in the fatigue test (Parts 2 and 3).
  - This explanation seems to us to be consistent with all the circumstantial evidence.
  - The only other defects found in the aeroplane (listed in Section 3) were not concerned at Elba, as demonstrated by the wreckage.
- Owing to the absence of wreckage, we are unable to form a definite opinion on the cause of the accident near Naples, but we draw attention to the fact that the explanation offered above for the accident at Elba appears to be applicable to that at Naples.

Accident Note 270 September 1954

Court of Inquiry – October 1954

- The Court of Inquiry agreed with the RAE findings
  - The crash of 'YP' was caused by fatigue failure of the pressure cabin
  - Similarities between 'YP' and 'YY' indicated that the cause was similar
    - First use of medical forensics to solve an air accident
- De Havilland was working at or beyond the limits of knowledge but had taken all precautions to prevent failure
- The 2P proof test probably plastically deformed any cracks allowing them to be less damaging in the subsequent fatigue tests

- *"Enough is now known about the fundamental physics of fatigue for engineers to be aware that there is still much to be learnt."*

Prof. AJ Murphy, 1954

De Havilland Comet I

- The Comet I was grounded until changes were made to the pressure cabin
  - Thicker skin around the windows
  - Altered shape to allow ‘Redux’ fixing
- Increased weight due to modifications
  - Not economic as a passenger transport
- Used by the RAE for investigations or the RAF as transport
- Comet 2 and Comet 3 development was stopped
  - RAF used the Comet 2 as transport after modifications

The story so far

- "Enough is now known about the fundamental physics of fatigue for engineers to be aware that there is still much to be learnt."

  Prof. AJ Murphy, 1954

- The general story is that this tragedy was caused by:
  - Fatigue failure of the airframe
  - Square windows were the cause of the high stresses

- This was all reported in the media

- ***What happened next?***

Subsequent Work

- RAE continued investigation into fatigue
  - G-ALYR was used in tank tests
- Continued the work conducted during the previous investigation
- Used the Comets no longer in service
- Further simulated flights in the tank test
- Cracks identified and monitored during the test
  - Some cracks stopped
  - A number grew to cause catastrophic failure
- Strain gauges used to measure the stress around the windows

Subsequent Work

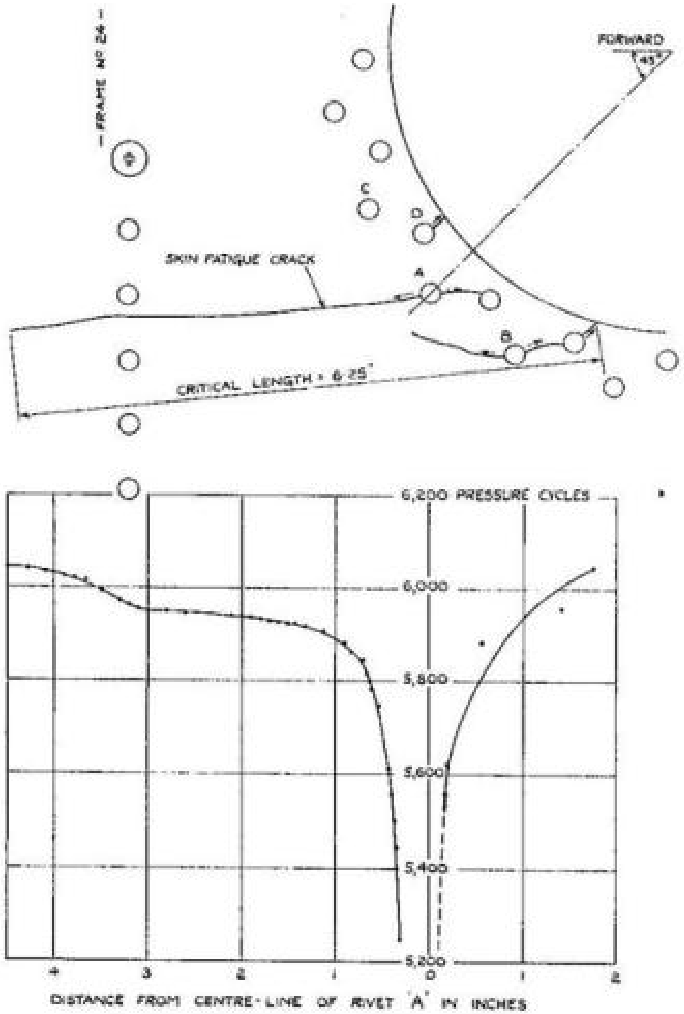

FIG. 21. Crack growth at third window, port side. (Bottom forward corner)

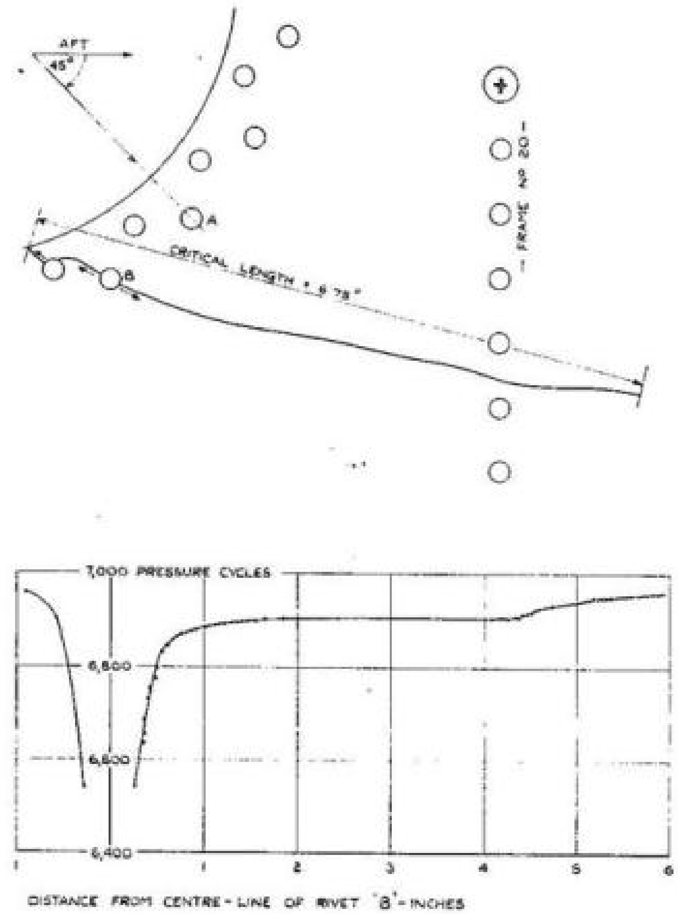

FIG. 22. Crack growth at first window, port side. (Bottom rear corner)

Subsequent Work

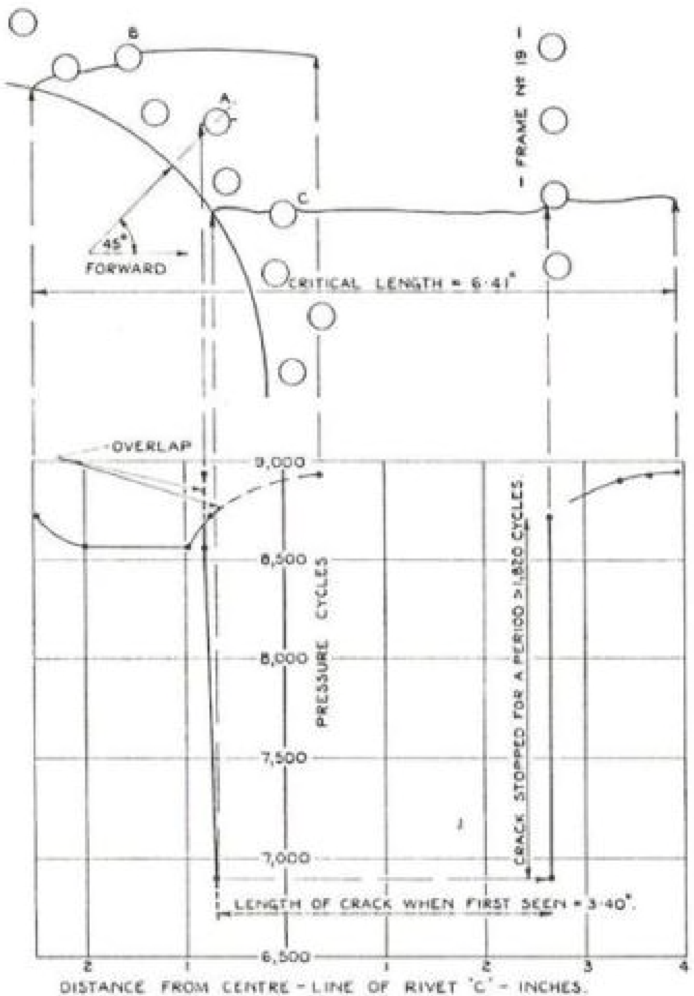

FIG. 25. Crack growth at first window, starboard side. (Top forward corner)

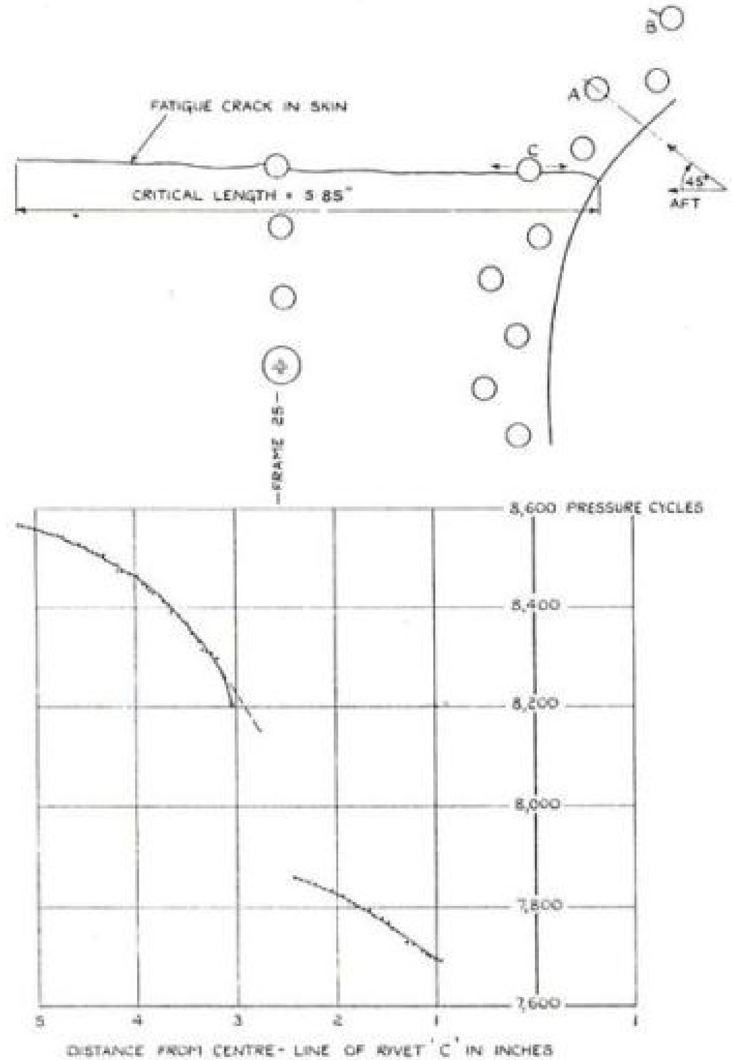

FIG. 26. Crack growth at third window, starboard side. (Top rear corner)

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Subsequent Work

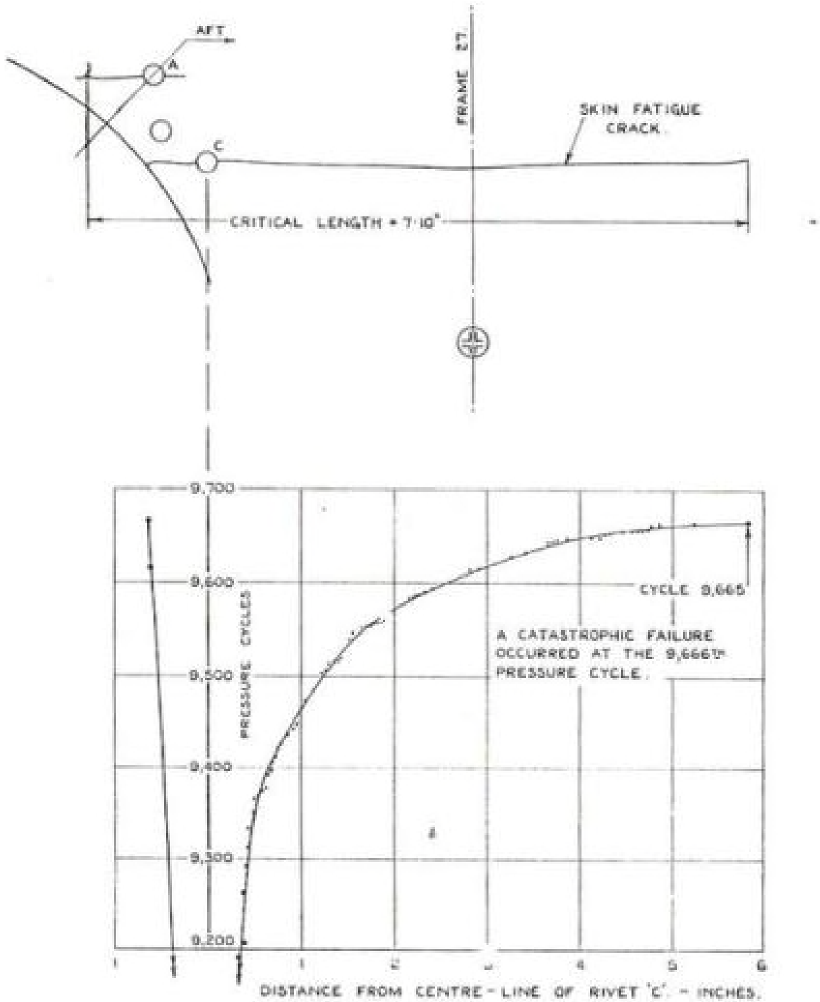


FIG. 29. Crack growth at fourth window, port side. (Top rear corner)

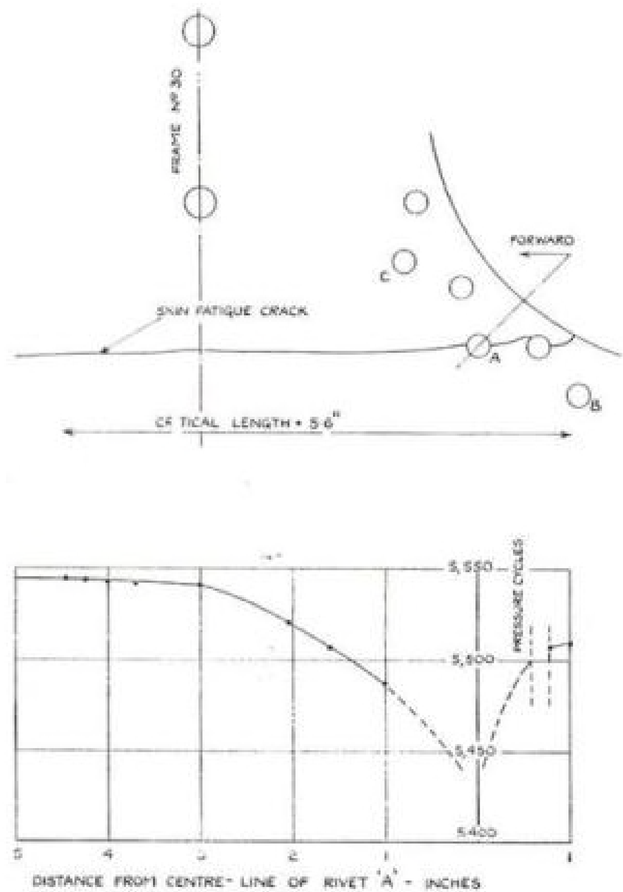
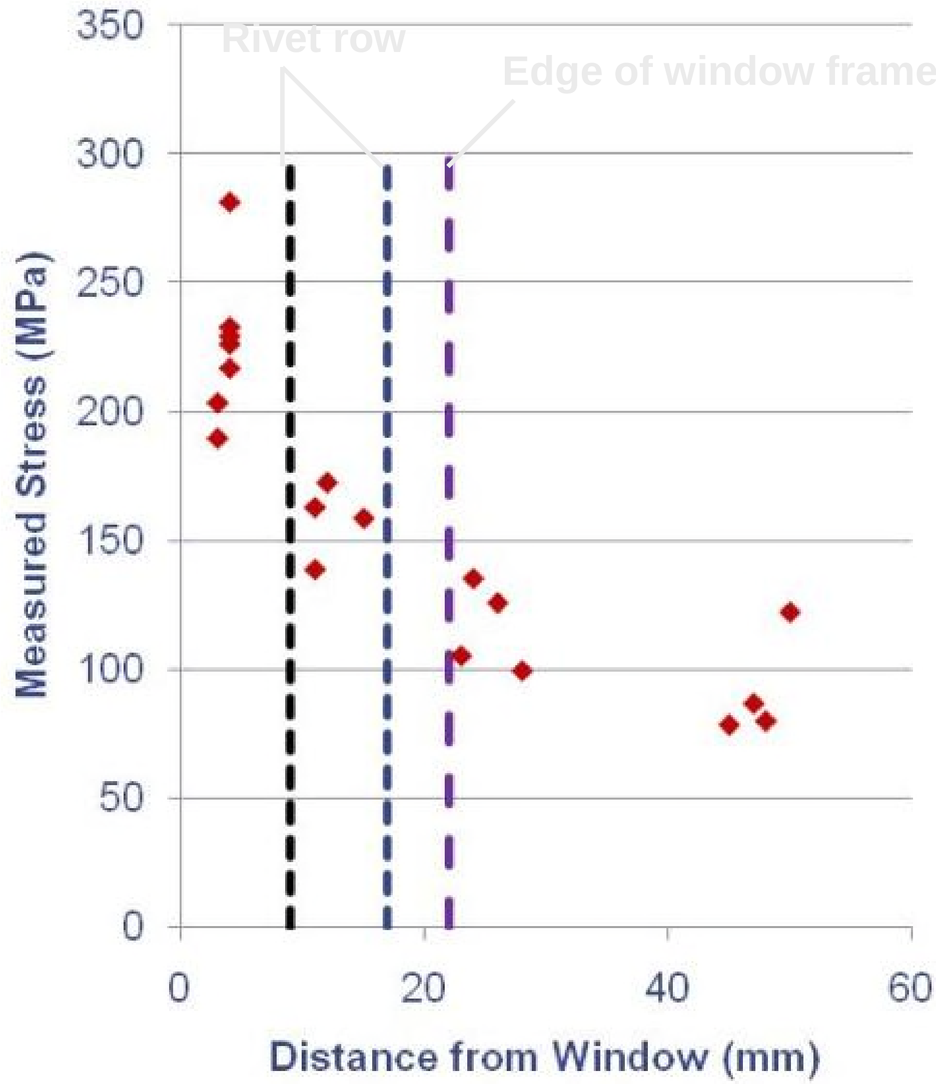


FIG. 30. Crack growth at No. 6 window, port side, in *Comet* G-ALYU. [*N.B.* The pressure cycles above include the 1231 pressurized flights made by this aircraft during service.]

Subsequent Work

- Strain gauge measurements looked at the stress around the cut outs
- Stresses agreed with previous work
- High level of stress near to cut outs
- Stress dropped quickly away from the cut outs



The Next Steps

- The understanding of fatigue developed over the coming years
- Focused effort on both the causes and the prediction of fatigue
- The first methods for relating fatigue crack growth rate to the instantaneous crack length and applied stress amplitude were published in 1961
- Paris and Erdogan proposed a method of predicting fatigue crack growth in 1963
- Empirical methods are applied to metals and alloys

- DTD546 superseded and not investigated further

Modern Analysis

- For an investigation today we could use the Paris Law
  - First published in 1963 by Paris and Erdogan
  - $da/dN = A\Delta K^m$
- From crack growth data calculate the Paris Exponent (m)
- Calculate the fracture toughness (K_{Ic}) from failure stress data
- With this data we could calculate
  - Initial crack sizes
  - Component lives

Modern Analysis

- For DTD546 there is little data
  - Not tested in this manner as it fell out of use

- What data do we have?

- Fatigue tests on ‘YU’ and ‘YR’
- Strain gauge data from these tests
- Evidence from the rebuild of ‘YP’

Modern Analysis

- Cracks grown to failure on Comet 'YR' in the water tank at the RAE
  - Data published showing loading cycles against crack growth

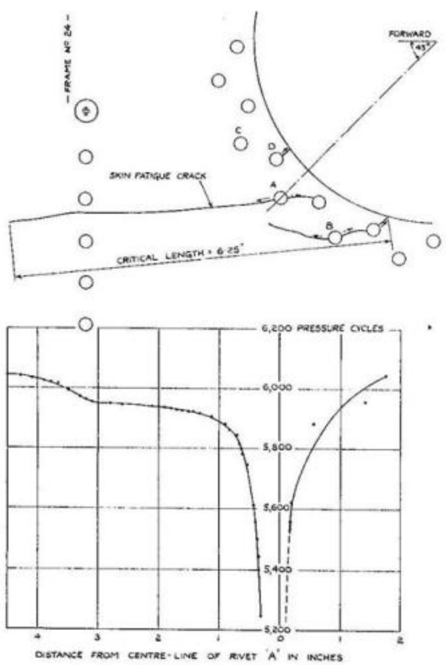


FIG. 21. Crack growth at third window, port side. (Bottom forward corner)

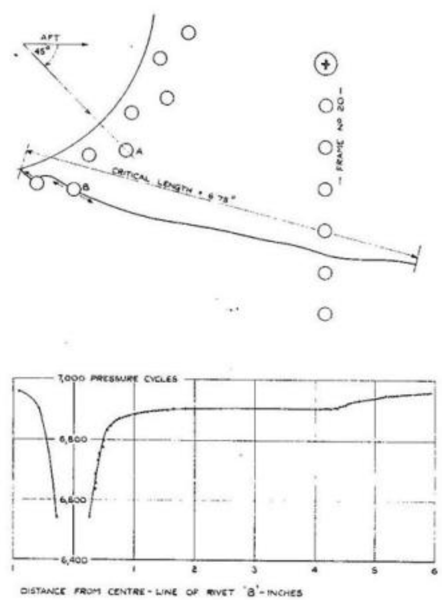


FIG. 22. Crack growth at first window, port side. (Bottom rear corner)

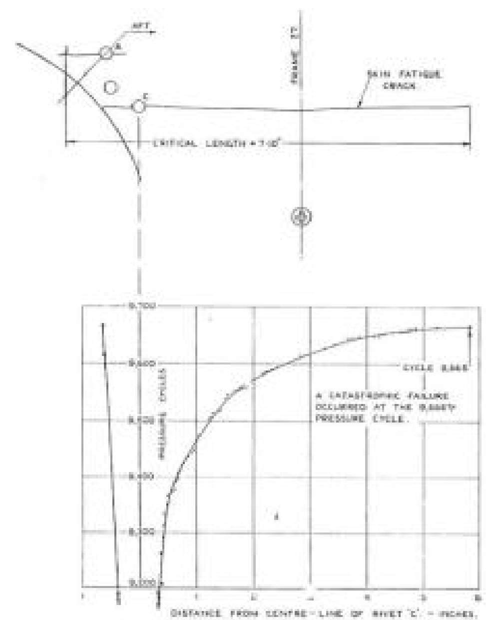
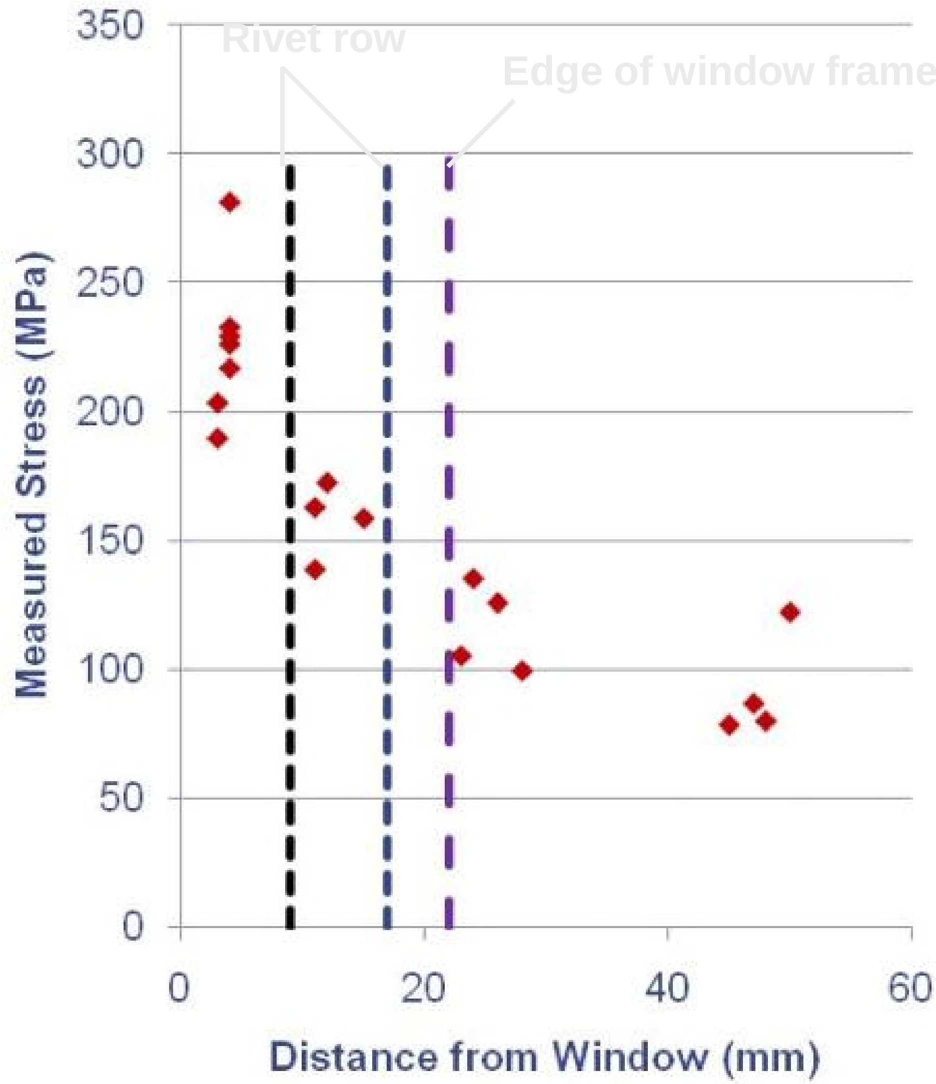


FIG. 29. Crack growth at fourth window, port side. (Top rear corner)

Modern Analysis

- Strain gauge measurements on 'YR' were also published
- Allows the stress range to be calculated for the various crack lengths



Modern Analysis

- Using the information on the Comet
  - Fuselage diameter 3.2 m
  - Skin thickness 0.71 and 0.91 mm
  - Cabin pressure of 59 kPa
- The hoop stress can be calculated
  - 128 MPa for the general skin 98 MPa for the thicker skin
- Slightly higher than the measured general stress of 69 MPa found during the RAE work
  - Possibly due to the ribs and stringers in the cabin
- Five cracks were allowed to grow to failure
  - Lengths between 149 mm and 180 mm
- Gives a calculated Fracture Toughness of 49.5 MPa√m

Modern Analysis – Fatigue Crack Growth Plot

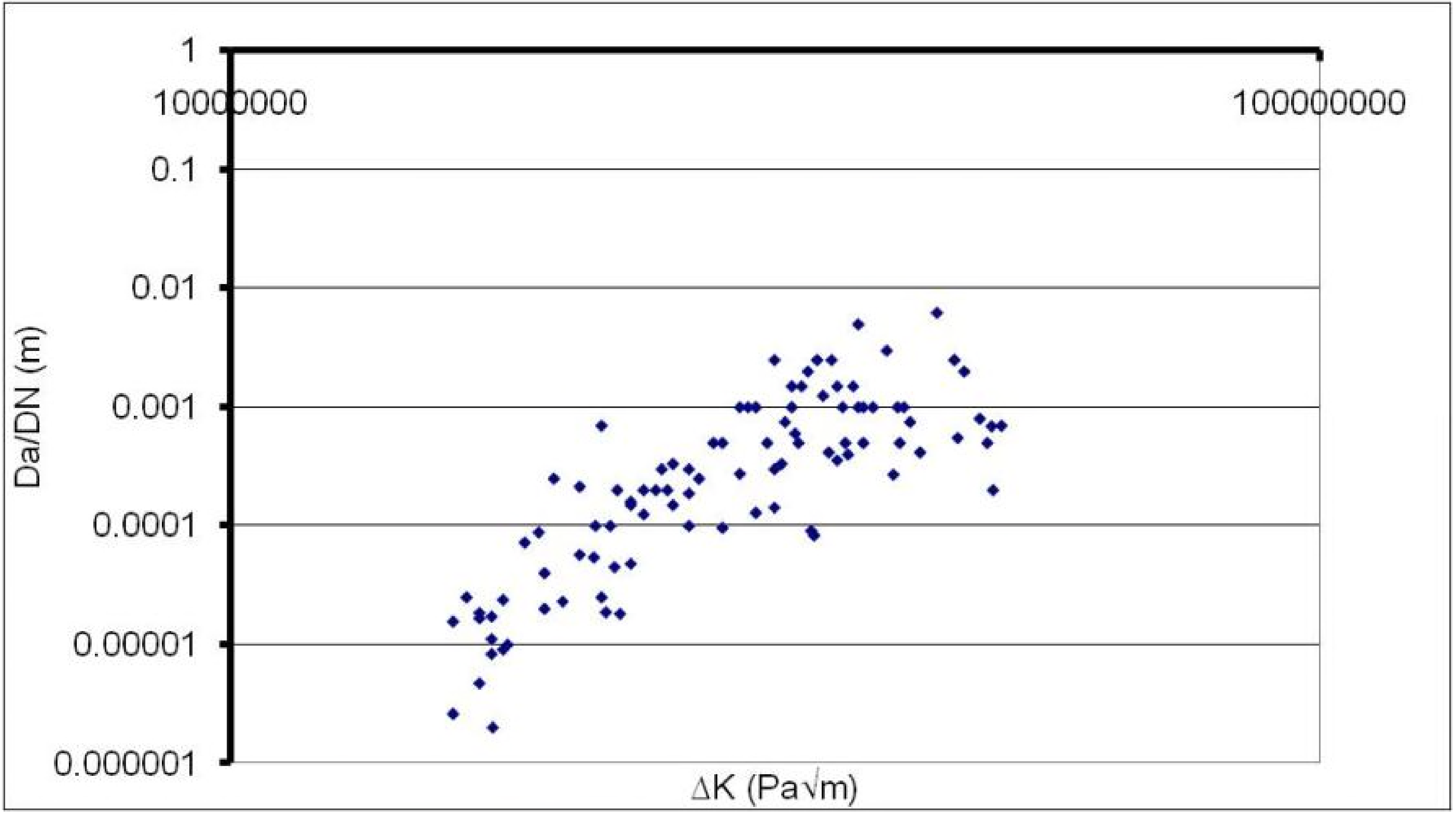
- Using the data available
- Calculate ΔK
  - $\Delta K = \Delta\sigma\sqrt{(\pi a)}$
  - a is the crack length
    - Treat as a single crack at a hole in an infinite sheet under biaxial loading (hoop and longitudinal stress)
  - $\Delta\sigma$ can be calculated from the measured stress in the tank tests
    - Varies with crack length

Modern Analysis – Fatigue Crack Growth Plot

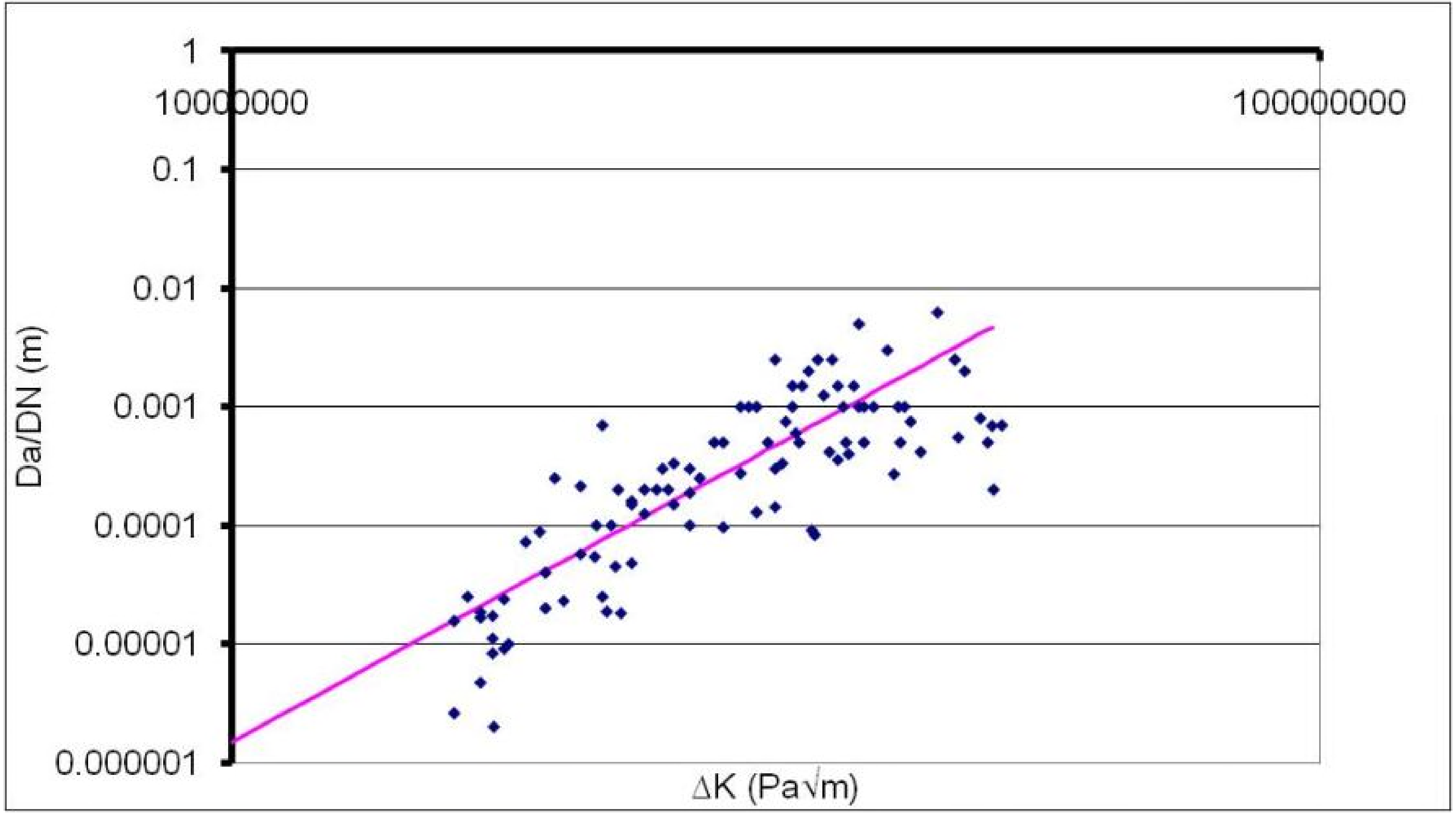
- Da/DN can be derived from the tank test Comet data
  - 'YU' and 'YR'

- Using Da/dN = $A\Delta K^m$
- We can take logs of both sides
  - log(Da/DN) = m log($A\Delta$K)
- Plot log (Da/dN) against logΔK
  - Straight line plot with a gradient of m

Modern Analysis – Fatigue Crack Growth Plot



Modern Analysis – Fatigue Crack Growth Plot



Analysis – Fatigue Crack Growth

- Using the data from the 1950's we can calculate that:
  - The Paris exponent (m) for the material is 5
  - Fracture toughness of around 49.5 MPa√m

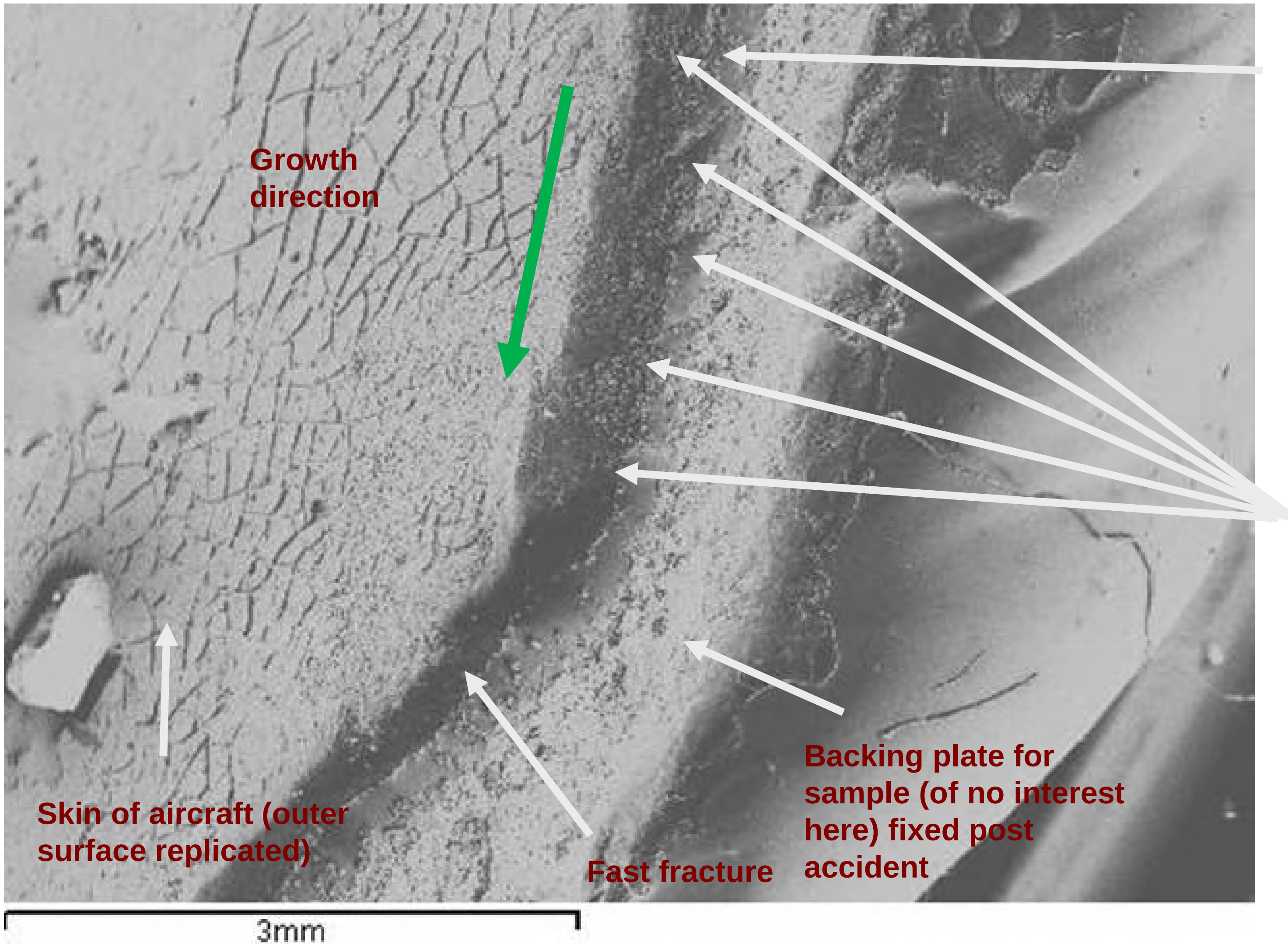
- The material behaved slightly worse than the current similar alloys but not remarkably so

- The initial cracks were likely to be very small (<1mm) and therefore difficult to identify on inspection
  - Initial manufacturing crack on 'YP' was likely to be hidden under the bolt head

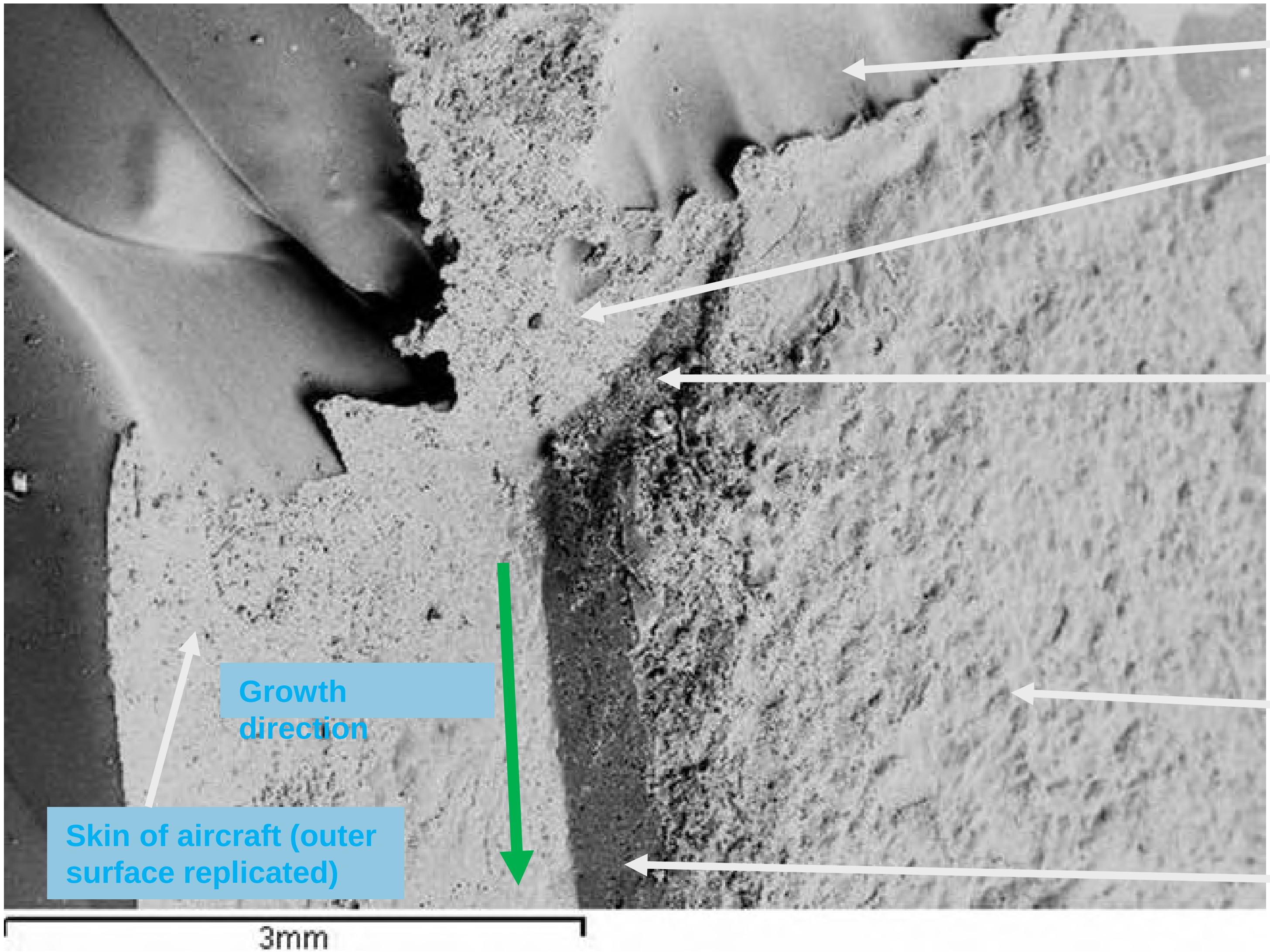
Modern Analysis – ‘YP’ Failure Site

Modern Analysis – ‘YP’ Failure Site

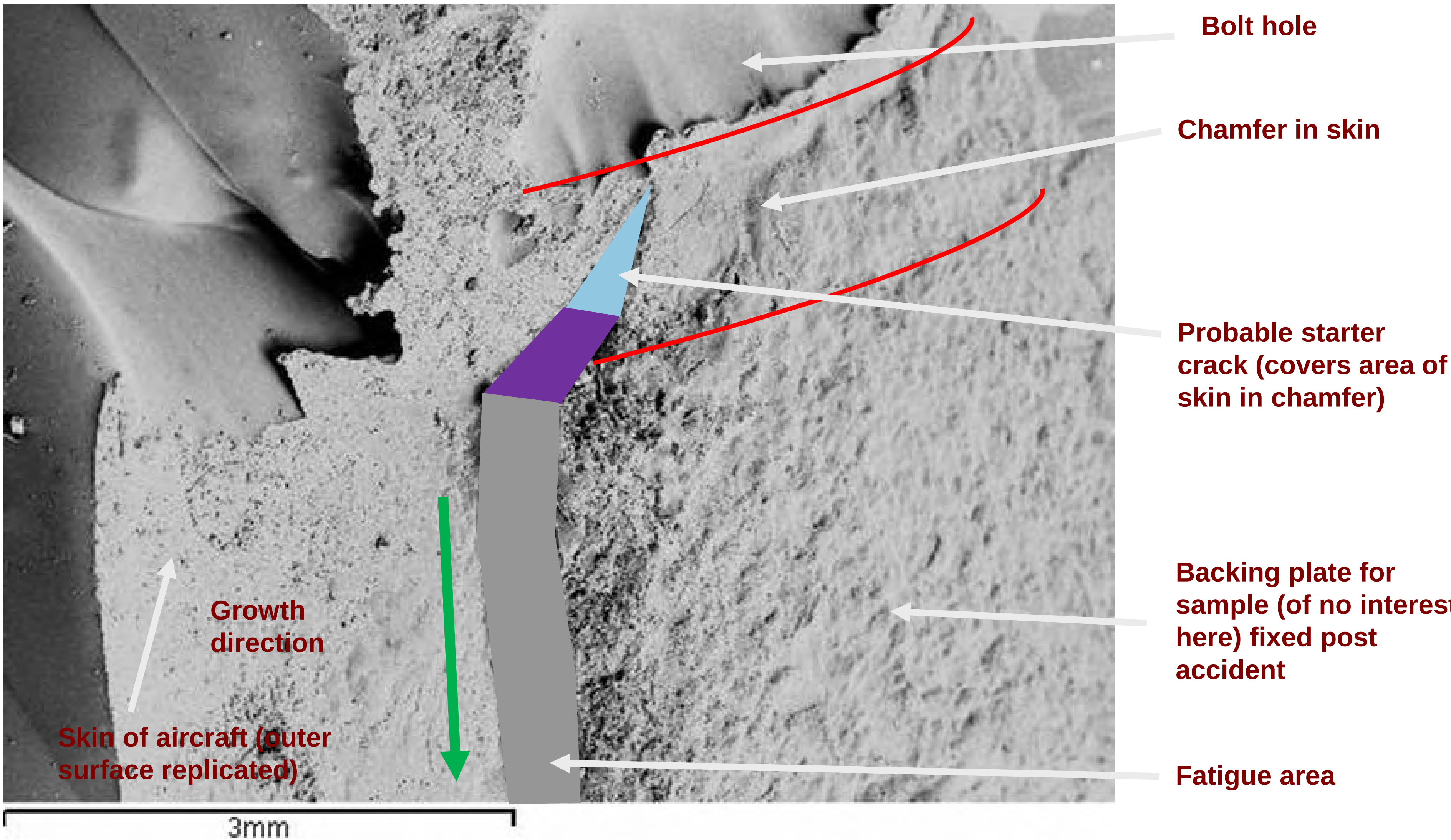
Fatigue Replica – SEM Analysis



Fatigue Replica – SEM Analysis

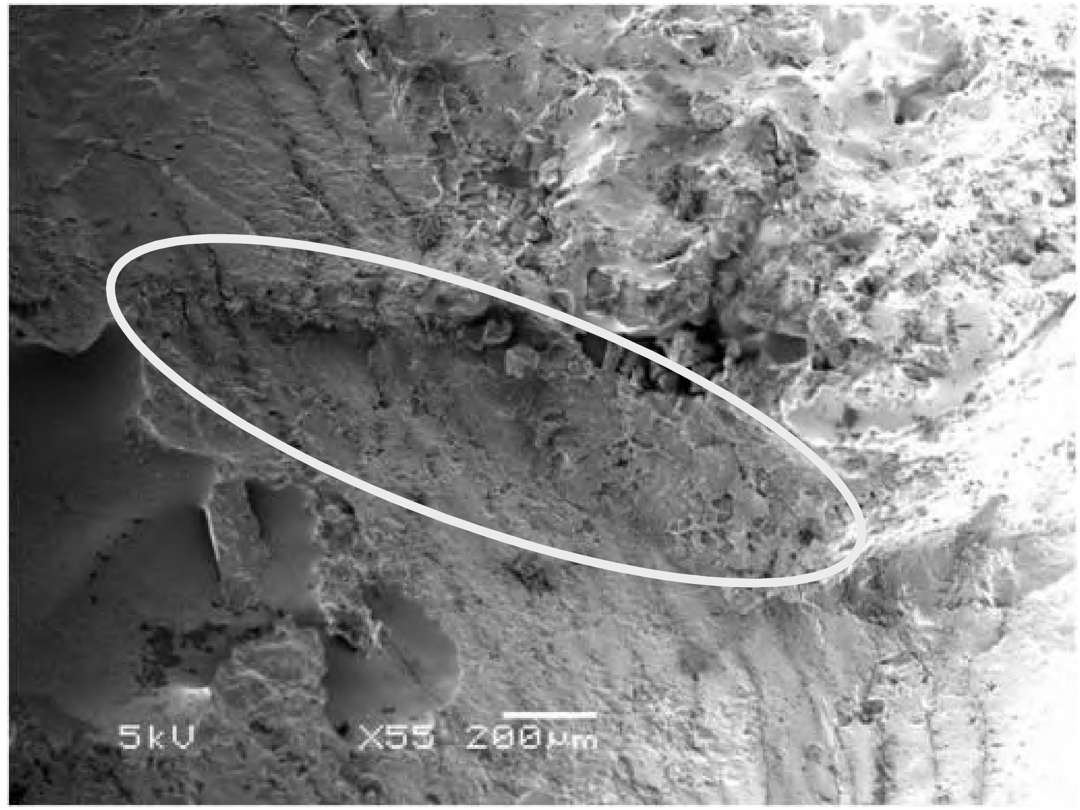


Fatigue Replica – SEM Analysis



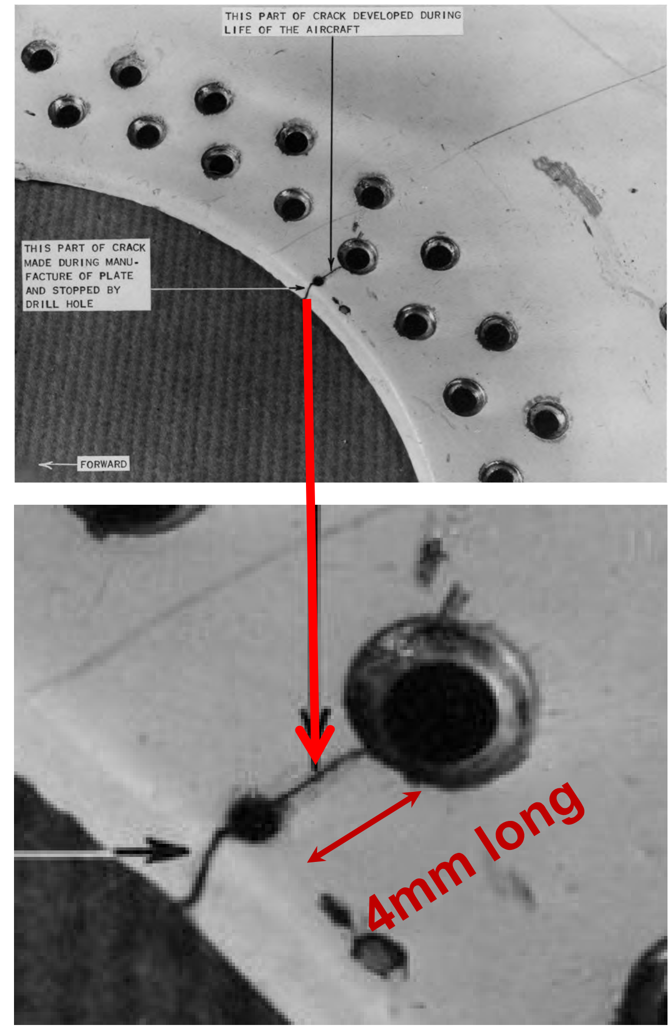
Fatigue Replica – SEM Analysis

- Initial defect extends from bolt hole around 1.5 mm
- Then turns in the direction of maximum stress



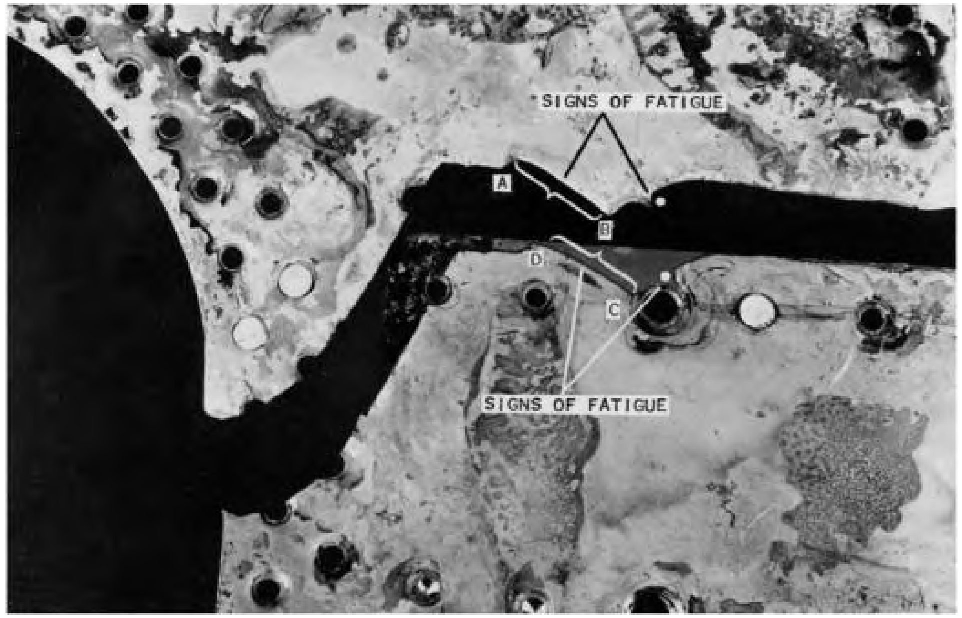
Modern Analysis

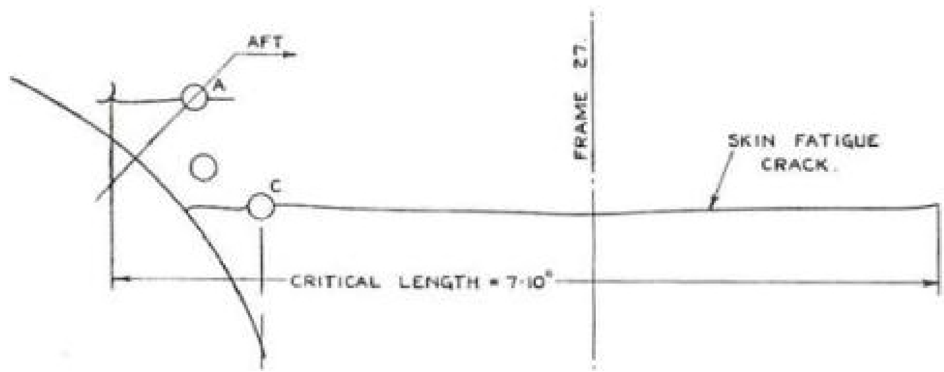
- Manufacturing cracks were present in the cabin skin
  - Rivets were pushed through causing cracks
  - Holes were drilled to ‘stop’ cracks
- Cracks could develop at the window rivets
  - Grew towards the window and stopped (seen on ‘YP’ and ‘YR’) (maximum crack length 20mm)
  - Grew away from the window into lower stress area and slowed down
  - Easy to spot and gave a cabin life of over 5000 cycles
- The critical crack length in a general area was around 165mm



Modern Analysis

- Bolt hole on ‘YP’ was around 90mm from the window
- Cracks grew both forward and aft from the bolt hole
- The cracks grew in the skin under a doubler plate and could not be seen
- The forward crack was growing into an area of increasing stress and therefore accelerated rapidly
- When the crack reached the ADF window it exceeded the critical crack length





Crack growth analysis

- If we start with the calculated Paris equation

$$\frac{da}{dN} = 1.5x10^{-41} \Delta K^{5}$$

- And substitute in for ΔK

$$\Delta K = \Delta\sigma(\pi a)^{1/2}$$

- We get

$$\frac{da}{dN} = 1.5x10^{-41} \Delta\sigma^{5} (\pi a)^{5/2}$$

Crack growth analysis

$$\frac{da}{dN} = 1.5x10^{-41}\Delta\sigma^{5}(\pi a)^{5/2}$$

- Rearranging to get

$$a^{-5/2}da = 1.5x10^{-41}\Delta\sigma^{5}\ \pi^{5/2}\ dN$$

- Then integrating

$$\frac{-2}{3}\left(a_f^{-3/2} - a_0^{-3/2}\right) = 1.5x10^{-41}\Delta\sigma^{5}\ \pi^{5/2}\ N_f$$

Crack growth analysis

- The hoop stress can be calculated to be 128 MPa
- The initial crack length was 1.5mm (from the SEM analysis)
- The final crack length on 'YP' was 165mm (from the total crack length)

- Feeding these numbers into the equation

$$\frac{-2}{3}\left(a_f^{-3/2} - a_0^{-3/2}\right) = 1.5x10^{-41} \Delta\sigma^5 \ \pi^{5/2} \ N_f$$

- The life of 'YP' can be calculated as 1272 cycles

- The actual number of pressurised flights was 1290.

Modern Analysis

- The failure of the pressure cabin was brought about by fatigue (In agreement with the 1954 findings)
- The manufacturing process of punch riveting (rather than drill, rivet and glue) caused small cracks which grew under the repeated loading of the pressure cabin
- The bolt hole which failed on ‘YP’ had a defect in the chamfer which indicated the potential for manufacturing defects on all skin holes
- The interaction of the skin stresses and the manufacturing defects was beyond the scientific knowledge base of the early 1950s

Comet 4

- The Comet flew again as the Comet IV
  - Different window design to reduce riveting
  - Fewer manufacturing cracks
- First airliner to fly a scheduled service across the Atlantic
  - 4th October 1958
- Remained in service as the Nimrod
  - 60 years after first Comet flight

Appendix: Introduction to "Paris' Law"

Paris' law

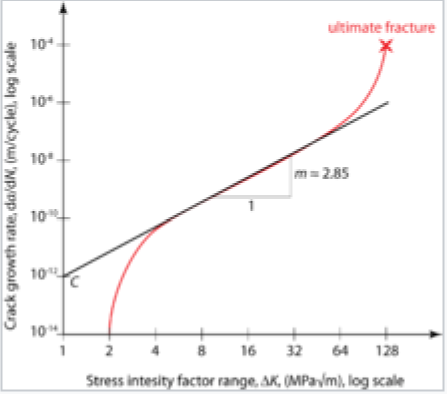
From Wikipedia, the free encyclopedia

Paris' law (also known as the **Paris-Erdogan law**) relates the stress intensity factor range to sub-critical crack growth under a fatigue stress regime. As such, it is the most popular *fatigue crack growth model* used in materials science and fracture mechanics. The basic formula reads[1]

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C\Delta K^m,$$

where a is the crack length and $\mathrm{d}a/\mathrm{d}N$ is the *crack growth rate*, [2] which denotes the crack growth for a load cycle. On the right hand side, C and m are constants that depend on the material, environment and stress ratio,[3] and ΔK is the range of the stress intensity factor during the fatigue cycle, i.e.,

$$\Delta K = K_{max} - K_{min}.$$ [3]



Schematic plot of the typical relationship between the crack growth rate and the range of the stress intensity factor. In practice, the Paris law is calibrated to model the linear interval around the center.

Paris examined a number of alloys and realised that plots of crack growth rate against range of stress intensity factor gave straight lines on log-log scales (see Figure). This implies that:

$$\log\left(\frac{da}{dN}\right) = m\log(\Delta K) + \log C$$

Taking out the logs gives:

$$\frac{da}{dN} = C\Delta K^m$$

da/dN is the increase of crack length for one load cycle (or the incrase of crack length from *N* load cycles). *N* is the number of load cycles.

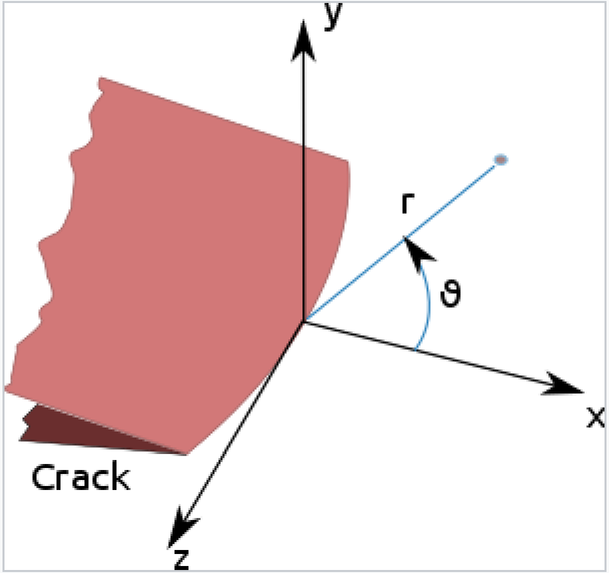
From: https://www.fose1.plymouth.ac.uk/fatiguefracture/tutorials/FractureMechanics/Fatigue/FatTheory1.htm

Stress intensity factor

From Wikipedia, the free encyclopedia

The **stress intensity factor**, K, is used in fracture mechanics to predict the stress state ("stress intensity") near the tip of a crack caused by a remote load or residual stresses.[1] It is a theoretical construct usually applied to a homogeneous, linear elastic material and is useful for providing a failure criterion for brittle materials, and is a critical technique in the discipline of damage tolerance. The concept can also be applied to materials that exhibit *small-scale yielding* at a crack tip.

The magnitude of K depends on sample geometry, the size and location of the crack, and the magnitude and the modal distribution of loads on the material.



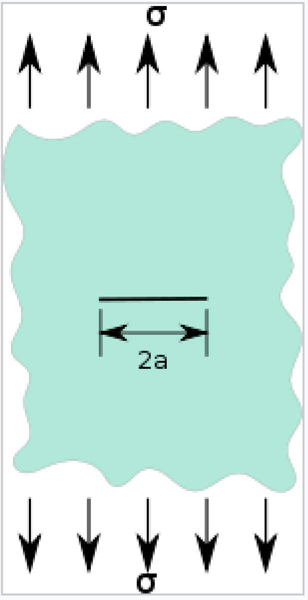
Polar coordinates at the crack tip.

Examples of stress intensity factors

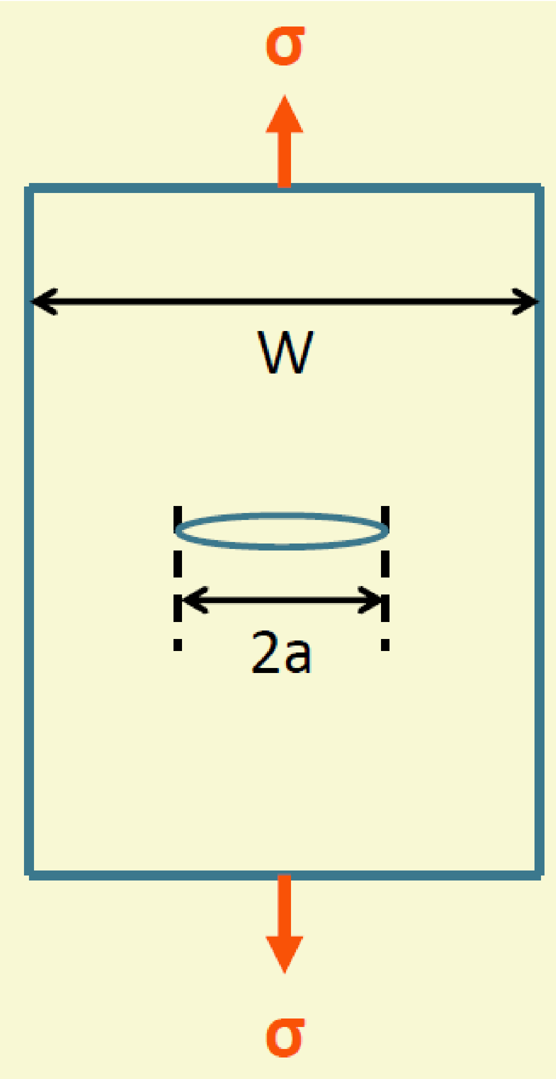
Infinite plate: Uniform uniaxial stress

The stress intensity factor for an assumed straight crack of length $2a$ perpendicular to the loading direction, in an infinite plane, having a uniform stress field σ is

$$K_{\mathrm{I}} = \sigma\sqrt{\pi a}$$



Crack in an infinite plate under mode I loading.



$$K = Y\sigma\sqrt{\pi a}$$

$$Y = \left\{\cos\left(\frac{\pi a}{W}\right)\right\}^{-\frac{1}{2}}$$

For small cracks with *a*/*W* → 0: *Y* = 1

From: https://www.le.ac.uk/eg/mct6/teaching/EG2101-L21a_RT2014.pdf

History and use

In a 1961 paper, P.C. Paris introduced the idea that the rate of crack growth may depend on the stress intensity factor.[4] Then in their 1963 paper, Paris and Erdogen indirectly suggested the **Paris law** with the aside remark "The authors are hesitant but cannot resist the temptation to draw the straight line slope 1/4 through the data..." after reviewing data on a log-log plot of crack growth versus stress intensity range.[5] The Paris equation was then presented with the fixed exponent of 4. Being a power law relationship between the crack growth rate during cyclic loading and the range of the stress intensity factor, the Paris law can be visualized as a linear graph on a log-log plot, where the x-axis is denoted by the range of the stress intensity factor and the y-axis is denoted by the crack growth rate.

Paris' law can be used to quantify the residual life (in terms of load cycles) of a specimen given a particular crack size. Defining the stress intensity factor as

$$K = \sigma Y \sqrt{\pi a},$$

where σ is a uniform tensile stress perpendicular to the crack plane and Y is a dimensionless parameter that depends on the geometry, the range of the stress intensity factor follows as

$$\Delta K = \Delta\sigma Y \sqrt{\pi a},$$

where $\Delta\sigma$ is the range of cyclic stress amplitude. Y takes the value 1 for a center crack of length $2a$ in an infinite sheet. The remaining cycles can be found by substituting this equation in the Paris law.

For relatively short cracks, *C* can be assumed as independent of *a* and the differential equation can be solved via separation of variables

$$\int_0^{N_f} \mathrm{d}N = \int_{a_i}^{a_c} \frac{\mathrm{d}a}{C(\Delta\sigma Y\sqrt{\pi a})^m} = \frac{1}{C(\Delta\sigma Y\sqrt{\pi})^m} \int_{a_i}^{a_c} a^{-\frac{m}{2}}\,\mathrm{d}a$$

and subsequent integration

$$N_f = \frac{2\left(a_c^{\frac{2-m}{2}} - a_i^{\frac{2-m}{2}}\right)}{(2-m)\,C(\Delta\sigma Y\sqrt{\pi})^m},$$

where N_f is the remaining number of cycles to fracture, a_c is the critical crack length at which instantaneous fracture will occur, and a_i is the initial crack length at which fatigue crack growth starts for the given stress range $\Delta\sigma$. If *Y* strongly depends on *a*, numerical methods might be required to find reasonable solutions.

References [edit]

1. ^ "The Paris law". *Fatigue crack growth theory*. University of Plymouth. Retrieved 28 January 2018.
2. ^ N. Pugno, M. Ciavarella, P. Cornetti, A. Carpinteri. "A generalized Paris' law for fatigue crack growth" (PDF). *Journal of the Mechanics and Physics of Solids 54 (2006) 1333-1349.*
3. ^ [a] [b] Roylance, David (1 May 2001). "Fatigue" (PDF). Department of Materials Science and Engineering, Massachusetts Institute of Technology. Retrieved 23 July 2010.
4. ^ P.C. Paris, M.P. Gomez, and W.E. Anderson. A rational analytic theory of fatigue. *The Trend in Engineering*, 1961, **13**: p. 9-14.
5. ^ P.C. Paris and F. Erdogen. A critical analysis of crack propagation laws. *Journal of Basic Engineering*, 1963.

Ref. 1: https://www.fose1.plymouth.ac.uk/fatiguefracture/tutorials/FractureMechanics/Fatigue/FatTheory1.htm
Ref. 2: http://www.ing.unitn.it/~pugno/NP_PDF/42-JMPS06.pdf
Ref. 3: https://ocw.mit.edu/courses/materials-science-and-engineering/3-11-mechanics-of-materials-fall-1999/modules/MIT3_11F99_fatigue.pdf

Paris Law integration (1)

- Paris' Law describes the steady-state crack growth typically seen under cyclic loading
- Definition: $\frac{\mathrm{d}a}{\mathrm{d}N} = A(\Delta K)^m$
- We can therefore calculate fatigue life by rearranging and integrating this relationship:
  - $N_f = \int_0^{N_f} \mathrm{d}N = \int_{a_0}^{a_f} \frac{\mathrm{d}a}{A(\Delta K)^m}$
- Ignoring the fact that $Y = f(a)$ (as it is often an empirical relationship):
  - $N_f = \int_{a_0}^{a_f} \frac{\mathrm{d}a}{A\left(Y\Delta\sigma\sqrt{\pi a}\right)^m} = \frac{1}{A\left(Y\Delta\sigma\sqrt{\pi}\right)^m} \int_{a_0}^{a_f} a^{-\frac{m}{2}}\, \mathrm{d}a$

Paris Law integration (2)

- Carry out the Paris Law integration:
  - $N_f = \frac{1}{A(Y\Delta\sigma\sqrt{\pi})^m}\int_{a_0}^{a_f} a^{-\frac{m}{2}}\,\mathrm{d}a$ (1)
- Show that you can obtain:
  - $N_f = \frac{1}{A(Y\Delta\sigma\sqrt{\pi})^m}\left[\frac{a^{-m/2+1}}{-m/2+1}\right]_{a_0}^{a_f}$
  - $N_f = \frac{1}{A(Y\Delta\sigma\sqrt{\pi})^m}\left(\frac{1}{-m/2+1}\right)\left[{a_f}^{-\frac{m}{2}+1} - {a_0}^{-\frac{m}{2}+1}\right]$ (2)
- This can be rearranged into a more convenient form:
  - $N_f = \frac{1}{A(Y\Delta\sigma\sqrt{\pi})^m}\left(\frac{1}{m/2-1}\right)\left[\frac{1}{{a_0}^{\frac{m}{2}-1}} - \frac{1}{{a_f}^{\frac{m}{2}-1}}\right]$ (3)

From: https://www.le.ac.uk/eg/mct6/teaching/EG2101-L21a_RT2014.pdf